LAYERED MATERIAL IDENTIFICATION USING MULTI-MODEL UPDATING

Tom Lauwagie[1] and Eddy Dascotte[2]

[1]University of Leuven - PMA
Celestijnenlaan 300B
3001 Leuven, Belgium
tom.lauwagie@mech.kuleuven.ac.be

[2] Dynamic Design Solutions (DDS) NV
Interleuvenlaan 64, B-3001 Leuven, Belgium
eddy.dascotte@dds.be, www.femtools.com

SUMMARY: Finite element model updating can be used to identify structural or material properties. In some cases the usable test data is very limited, while the number of variables can potentially be very large, leading to underdetermined systems of equations. Moreover, the system of equations can be ill-conditioned. Therefore, the identification results benefit from an increased number of reference test data sets. For this purpose, a method has been developed to simultaneously update multiple finite element models using their corresponding test data. The updating variables in each model must be properties that are common to all models while the other properties can be different in every model. Examples of properties that can be common are material properties and joint stiffness. Examples of properties that can be different in each model are lumped mass, boundary conditions, and shape. The number of models taken into account is only limited by practical considerations. The general purpose Multi-Model Updating (MMU) procedure is explained and demonstrated to identify a layered laminated material using different test specimen. It is shown that this type of model updating leads to more reliable identification results.

KEYWORDS: Model Updating, Multi-Model Updating, Material Identification.

INTRODUCTION

During the past decennium, finite element (FE) model updating has been an important research topic in the field of structural dynamics. The research has resulted in many different techniques, mainly characterized by the choice of reference test data and updating parameters, and the parameter estimation algorithm. Most commonly used are the sensitivity-based iterative methods because they provide the highest level of control on the choice of updating parameters and targets and allowing physical interpretation of the updating results. Although the main application of model updating is still validation, calibration and improvement of simulation models, research projects are now going in the direction of new applications like structural health monitoring, damage detection and material identification in which model updating serves are a necessary component.

One of the major obstacles to successful industrial applications is the unavailability of a sufficient amount of good quality test data. In structural dynamics, test data are usually modally and spatially truncated, and raw test data is often polluted by noise leading to a situation where only data like resonance frequencies can be used with high confidence. As a result, the model updating procedure requires the solution of a highly underdetermined system of equations, i.e. the target responses can be obtained via an infinite number of combinations of updating parameter changes. However, applications like material identification require a unique solution. The only possible ways to overcome this problem are either to reduce the number of updating parameters (e.g. using error localization techniques and grouping) or to increase the amount of information in the reference test data. In

addition, weighing of the updating parameters and reference test data can be used to bias the solution to the most likely one i.e. the one in which the analyst has the most confidence.

Increasing the amount of test data is possible by including mode shape data (in addition to resonance frequencies) or by using Frequency Response Functions (FRFs). However, this will put a higher weight on the quality of the raw test data, or lead to numerical problems due to redundancy, or sensitivity scaling. An alternative approach that is discussed in this paper, is to use multiple sets of reference test data obtained from a set of different or modified structures.

Using perturbed boundary condition testing (PBC) as a way to obtain multiple sets of modal test data for the same structure has been explored by Brown et al. [1-4]. Although this technique was initially intended as a tool to derive a more robust model by using averaging and data condensation, its value for model updating was also demonstrated. With PBC, simple modifications are made to the structure so that it is possible to re-test without changing the test set-up. To be successful, however, it is required that the modifications are sufficiently important. This leads to the new problem of effective PBC test design. Changing boundary conditions is a more effective way to influence the mode shapes, but is more difficult to realize in practice (free/free testing is still preferred). A type of perturbation that is easier to realize in practice, but often with less impact on the mode shapes, is adding or removing lumped masses [5]. Another approach is to perform the model updating using forced vibration test data with different loads [6], since each load case yields a different set of test data. Ibrahim explored the use of multiple perturbed analytical models [7] and one single set of test data to improve the conditioning of the updating equations.

A generic procedure for simultaneously updating multiple FE models was recently implemented in the commercial FEMtools software for test-analysis correlation and model updating [8]. Using this multi-model updating (MMU) procedure, FE models of structures that were tested with different boundary conditions or lumped masses (like PBC), as well as FE models of different, even unrelated, structures with common physical properties, can be handled. The updating variables in each model must be physical properties that are common to all models while the properties that are not updated can be different for every model. Typical examples of properties that can be common to different models are material properties and joint stiffness. Examples of properties that may be different in each model are lumped mass, boundary conditions, and shape. However, it would also be possible to update the boundary condition properties (for example clamping stiffness) using modal test data from different structures with different geometry or material as long as they are constrained in the same way. The number of models taken into account is only limited by practical considerations.

Some examples of industrial cases that illustrate the different types of multi-model updating that can be handled using this procedure:
   (i)   a launch vehicle tested with different levels of fuel. The fuel is modelled using varying lumped masses while the launcher itself is a structure that is common in all tests, and thus can be updated.
   (ii)   satellite solar panels tested during different stages of deployment. In this case the structure is the same in all tests, no physical properties are modified. However, the geometric configuration of the structure is modified and hence the modal parameters that are measured are different with each stage of deployment. This provides a rich set of test data to update e.g. the joint stiffness.
   (iii)   different test specimens made of a layered (composite) material that needs to be identified. The material properties of the laminate or some layers are common to each test specimen and their estimated properties can be updated. Each test specimen can differ in shape or even laminate lay-up.

The example of layered material identification will be further described in this paper.

MULTI-MODEL UPDATING PROCEDURE

To use the MMU procedure that has been implemented in FEMtools [8], the analyst must first create a database of models that for each tested structure, contains the finite element model of the structure (as tested), the test model and data, tables relating the common degrees of freedom and the modal parameters in each model, and selections of updating targets and updating variables. This preparatory work is similar as if each model would be updated separately. For MMU, however, it is critical that the updating variables are of the same type and level in

each model. Next, the MMU control script takes over and performs updating of all the models in the database, using the following procedure:

1. Computation of the sensitivity matrix $\left[S_{ij}^k \right]$ for each of the k models in order to construct the following equation:

$$\left\{ \Delta R_i^k \right\} = \left[S_{ij}^k \right] \left\{ \Delta P_j \right\}$$

where $\left\{ \Delta R_i^k \right\}$ are the i residual differences between the predicted and measured responses for model k, and $\left\{ \Delta P_j \right\}$ are the j selected updating parameters common to the k considered models. Note that the number and the type of responses can be different in each model.

2. Assemblage of a global sensitivity matrix and solving for unknown $\left\{ \Delta P_j \right\}$ of

$$\begin{Bmatrix} \Delta R_i^1 \\ \Delta R_i^2 \\ \dots \\ \Delta R_i^k \end{Bmatrix} = \begin{bmatrix} S_{ij}^1 \\ S_{ij}^2 \\ \dots \\ S_{ij}^k \end{bmatrix} \left\{ \Delta P_j \right\}$$

3. Introduction of the resulting parameter changes $\left\{ \Delta P_j \right\}$ into each model and re-computation of the modal parameters of each model.

4. This procedure is repeated until a convergence criterion is satisfied.

This is a general applicable procedure that re-uses the existing tools for correlation analysis, sensitivity analysis and model updating. It offers maximum flexibility in the choice of updating targets that best fit each individual model.

EXAMPLE: IDENTIFICATION OF LAYERED MATERIALS

Introduction

Elastic properties play a major role in the determination of the vibration behaviour of structures and material samples. This observation can be turned around, leading to the conclusion that the elastic material properties can be derived from the vibration behaviour of test samples. For simple cases, the vibration behaviour of the test specimen can be described by analytical formulas based on the Timoshenko beam theory. Various methods based on these formulas exist, and have proven to produce repeatable and accurate results. Unfortunately, the use of these analytical methods is limited to the case of homogeneous isotropic materials.

The finite element technique allows the modelling of the vibration behaviour of test samples with an arbitrary shape combined with the use of more complex material models. In 1986 Sol [9] introduced an inverse method that could identify the properties of an orthotropic plate, by comparing calculated resonance frequencies of a finite element model of the test plate with experimentally measured frequencies. The possibilities to extend this inverse methods to identify the elastic properties of the individual layers of layered materials are currently being investigated. The first results of a preliminary study are promising, but highlight the necessity of a multi-model updating routine.

Material identification by model updating

Figure 1 presents the general flowchart of the inverse method to identify elastic material properties. Initial values for the elastic material properties are estimated and introduced into the FE model of the test plate in order to compute the numerical resonance frequencies and frequency sensitivities of the first five modes.
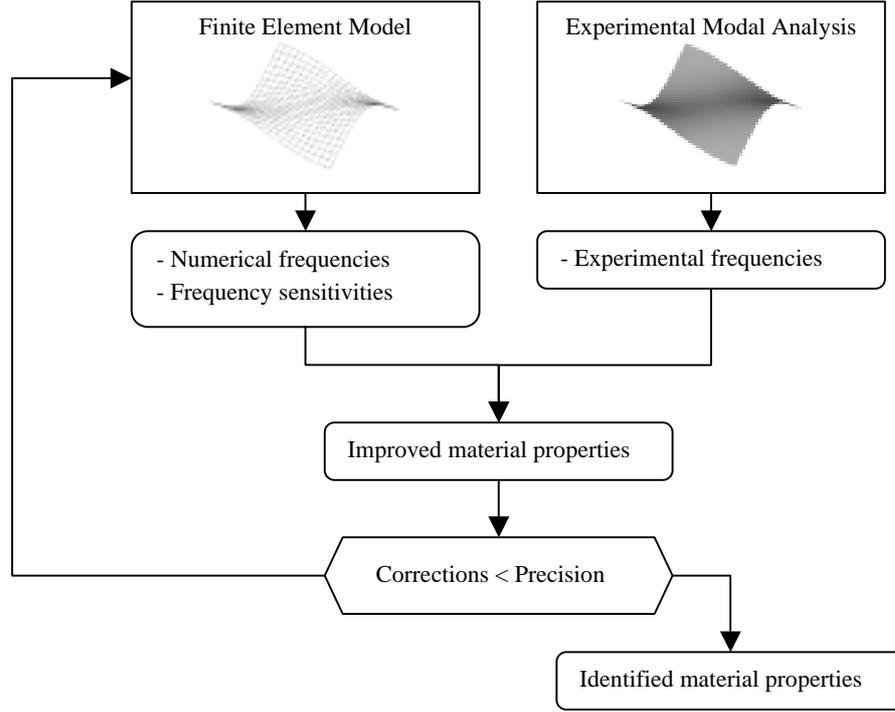
Figure 1: Flowchart of the inverse method for material identification.

Improved material properties can be obtained from the differences between the experimental and numerical frequencies and frequency sensitivities by solving the following least-squares problem:

$$\{\Delta p\} = [S_n]^\dagger \{\Delta f\} \tag{1}$$

in which

$$\Delta p = \frac{p_{new} - p_{old}}{p_{old}}, \ \Delta f = \frac{f_{exp} - f_{FE}}{f_{exp}}, \ and$$

$$S_n = \begin{bmatrix} \dfrac{\partial f_1}{\partial E_x} \dfrac{E_x}{f_1} & \dfrac{\partial f_1}{\partial E_y} \dfrac{E_y}{f_1} & \dfrac{\partial f_1}{\partial G_{xy}} \dfrac{G_{xy}}{f_1} & \dfrac{\partial f_1}{\partial \nu_{xy}} \dfrac{\nu_{xy}}{f_1} \\ \dfrac{\partial f_2}{\partial E_x} \dfrac{E_x}{f_2} & \dfrac{\partial f_2}{\partial E_y} \dfrac{E_y}{f_2} & \dfrac{\partial f_2}{\partial G_{xy}} \dfrac{G_{xy}}{f_2} & \dfrac{\partial f_2}{\partial \nu_{xy}} \dfrac{\nu_{xy}}{f_2} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial f_5}{\partial E_x} \dfrac{E_x}{f_5} & \dfrac{\partial f_5}{\partial E_y} \dfrac{E_y}{f_5} & \dfrac{\partial f_5}{\partial G_{xy}} \dfrac{G_{xy}}{f_5} & \dfrac{\partial f_5}{\partial \nu_{xy}} \dfrac{\nu_{xy}}{f_5} \end{bmatrix} \tag{2}$$

and where p denotes a material property, f denotes a resonance frequency and $\dagger$ denotes the pseudo inverse of a matrix.

Note that the sensitivity matrix S_n in equation (2) is composed with normalised relative sensitivities. There are two distinct reasons to prefer normalised relative sensitivities over classical absolute sensitivities. Absolute sensitivities of the frequencies with respect the Young's modulus vary a few orders of magnitude from those of Poisson's ratio, while this is not the case for relative sensitivities. Therefor the sensitivity matrix with relative sensitivities will be better conditioned, leading to an increased stability of the updating routine. By normalising the sensitivities, i.e. dividing them by the frequency, the least-squares solution (1) of the updating routine will minimise the relative differences between the numerical and experimental frequencies. If absolute sensitivities were being used, the least-squares routine would minimise the absolute frequency differences, giving the high order modes indirectly a higher weight.

If the corrections of the material properties resulting from (1) are larger than the desired precision, the improved material properties are introduced into the FE-model and a new iteration is started, otherwise the updating routine is aborted. If the frequencies of the FE-model of the last iteration match the experimental frequencies, the material properties of the test plate have been identified since they correspond with the material properties of the FE-model.

The necessity of a multi-model updating routine for identification of layered materials

The resonance frequencies of a layered test sample are controlled by the overall stiffness of the sample, and do not contain any information about the contribution of each of the individual layers. Even with a fixed geometry of the layers, there is a infinite number of different sets of material properties for the individual layers that result in the same global stiffness, and thus in the same resonance frequencies for the sample. Therefor the solution of an inverse identification procedure to identify the elastic properties of the layers from the resonance frequencies of one single sample will not be unique, and thus meaningless.

This conclusion can be verified by looking at the sensitivity matrix of the inverse problem. The structure of the sensitivity matrix of is presented in equation (3). Note that not the number of layers, but the number of materials that are used in the different layers determines the number of unknown parameters, therefor the number of block columns of the sensitivity matrix equals the number of used materials and not the number of material layers.

$$
S_n = \begin{bmatrix}
\overbrace{\dfrac{\partial f_1}{\partial E_{x,1}}\dfrac{E_{x,1}}{f_1}}^{\text{Material 1}} & \dfrac{\partial f_1}{\partial E_{y,1}}\dfrac{E_{y,1}}{f_1} & \dfrac{\partial f_1}{\partial G_{xy,1}}\dfrac{G_{xy,1}}{f_1} & \dfrac{\partial f_1}{\partial \nu_{xy,1}}\dfrac{\nu_{xy,1}}{f_1} & & \overbrace{\dfrac{\partial f_1}{\partial E_{x,n}}\dfrac{E_{x,n}}{f_1}}^{\text{Material }n} & \dfrac{\partial f_1}{\partial E_{y,n}}\dfrac{E_{y,n}}{f_1} & \dfrac{\partial f_1}{\partial G_{xy,n}}\dfrac{G_{xy,n}}{f_1} & \dfrac{\partial f_1}{\partial \nu_{xy,n}}\dfrac{\nu_{xy,n}}{f_1} \\
\dfrac{\partial f_2}{\partial E_{x,1}}\dfrac{E_{x,1}}{f_2} & \dfrac{\partial f_2}{\partial E_{y,1}}\dfrac{E_{y,1}}{f_{2,1}} & \dfrac{\partial f_2}{\partial G_{xy,1}}\dfrac{G_{xy,1}}{f_2} & \dfrac{\partial f_2}{\partial \nu_{xy,1}}\dfrac{\nu_{xy,1}}{f_2} & \cdots & \dfrac{\partial f_2}{\partial E_{x,n}}\dfrac{E_{x,n}}{f_2} & \dfrac{\partial f_2}{\partial E_{y,n}}\dfrac{E_{y,n}}{f_{2,1}} & \dfrac{\partial f_2}{\partial G_{xy,n}}\dfrac{G_{xy,n}}{f_2} & \dfrac{\partial f_2}{\partial \nu_{xy,n}}\dfrac{\nu_{xy,n}}{f_2} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\
\dfrac{\partial f_m}{\partial E_{x,1}}\dfrac{E_{x,1}}{f_m} & \dfrac{\partial f_m}{\partial E_{y,1}}\dfrac{E_{y,1}}{f_m} & \dfrac{\partial f_m}{\partial G_{xy,1}}\dfrac{G_{xy,1}}{f_m} & \dfrac{\partial f_m}{\partial \nu_{xy,1}}\dfrac{\nu_{xy,1}}{f_m} & & \dfrac{\partial f_m}{\partial E_{x,n}}\dfrac{E_{x,n}}{f_m} & \dfrac{\partial f_m}{\partial E_{y,n}}\dfrac{E_{y,n}}{f_m} & \dfrac{\partial f_m}{\partial G_{xy,n}}\dfrac{G_{xy,n}}{f_m} & \dfrac{\partial f_m}{\partial \nu_{xy,n}}\dfrac{\nu_{xy,n}}{f_m}
\end{bmatrix} \quad (3)
$$

Equation (4) gives the sensitivity matrix for the first ten frequencies of a symmetric double coated steel plate. It can be seen that the first column, containing the E_x sensitivities of the coating, and fifth column that contains the E_x sensitivities of the substrate, are linearly dependent. The second, third, and fourth column are respectively linearly related with the sixth, seventh and eight column.

$$
S_n = \begin{bmatrix}
\overbrace{\begin{matrix} E_x \\ 0.001285 \end{matrix}}^{\text{Coating}} & \overbrace{\begin{matrix} E_y \\ 0.001795 \end{matrix}} & \overbrace{\begin{matrix} G_{xy} \\ 0.036078 \end{matrix}} & \overbrace{\begin{matrix} \nu_{xy} \\ 0.000511 \end{matrix}} & \overbrace{\begin{matrix} E_x \\ 0.012777 \end{matrix}}^{\text{Substrate}} & \overbrace{\begin{matrix} E_y \\ 0.017803 \end{matrix}} & \overbrace{\begin{matrix} G_{xy} \\ 0.430262 \end{matrix}} & \overbrace{\begin{matrix} \nu_{xy} \\ 0.005064 \end{matrix}} \\
0.025625 & 0.020052 & 0.000054 & -0.004770 & 0.254702 & 0.198927 & 0.000642 & -0.047320 \\
0.018194 & 0.027423 & 0.000115 & 0.008350 & 0.180858 & 0.272046 & 0.001367 & 0.082809 \\
0.017445 & 0.002161 & 0.022093 & 0.001343 & 0.173376 & 0.021441 & 0.263484 & 0.013318 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0.013430 & 0.019987 & 0.010429 & 0.005723 & 0.133495 & 0.198283 & 0.124378 & 0.056750
\end{bmatrix} \quad (4)
$$

The relation between the different columns is further illustrated by Figure 2, in which each dot represents the E_x sensitivity for one particular frequency.
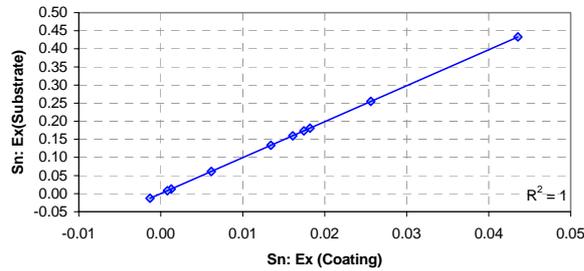

Figure 2: Relation between the first and fifth column of the sensitivity matrix.

The linear dependency of the columns of the sensitivity matrix can be interpreted in the following way. A variation of $E_{x,coating}$ will change the different frequencies in the same way as a variation of $E_{x,substrate}$. Therefore, a change of $E_{x,coating}$ can be completely compensated by an appropriate adaptation of $E_{x,substrate}$, and the FE-model with the adapted material properties will have exactly the same resonance frequencies as the initial FE-model. Eventually, this mechanism will result in an infinite number of possible solutions for the inverse problem.

The non-uniqueness of the solution can be solved by using a frequency set measured on a number of test samples with a different layer geometry. In each of the frequencies of such a set, the contribution of the different layers will be different, making it possible to separate the measured overall stiffness into the desired layer stiffness. In the case of layered materials, the single model updating routine of the standard material identification procedure has to be replaced by a multi-model updating routine.

The multi-model updating procedure for identification of layered materials

To guarantee the uniqueness of the solution, a set of specimen with different layer geometry must be tested, and the inverse material identification method must be a multi-model updating procedure. The global sensitivity matrix of this multi-model procedure is presented in equation (5), and contains one block column for every different material used in the samples, and one block row for every tested sample (or FE model). The improved material properties are still obtained with equation (1).

$$S_n = \begin{bmatrix} model-1 \begin{cases} \overbrace{\dfrac{\partial f_{1,1}}{\partial E_{x,1}} \dfrac{E_{x,1}}{f_{1,1}}}^{Material-1} & \cdots & \dfrac{\partial f_{1,1}}{\partial v_{xy,1}} \dfrac{v_{xy,1}}{f_{1,1}} & & \cdots & & \overbrace{\dfrac{\partial f_{1,1}}{\partial E_{x,n}} \dfrac{E_{x,n}}{f_{1,1}}}^{Material-n} & \cdots & \dfrac{\partial f_{1,1}}{\partial v_{xy,n}} \dfrac{v_{xy,n}}{f_1} \\ \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ \dfrac{\partial f_{m,1}}{\partial E_{x,1}} \dfrac{E_{x,1}}{f_{m,1}} & \cdots & \dfrac{\partial f_{m,1}}{\partial v_{xy,1}} \dfrac{v_{xy,1}}{f_{m,1}} & & \dfrac{\partial f_{m,1}}{\partial E_{x,n}} \dfrac{E_{x,n}}{f_{m,1}} & \cdots & \dfrac{\partial f_{m,1}}{\partial v_{xy,n}} \dfrac{v_{xy,n}}{f_{m,1}} \end{cases} \\ \vdots \qquad \qquad \ddots \qquad \qquad \vdots \\ model-k \begin{cases} \dfrac{\partial f_{1,k}}{\partial E_{x,1}} \dfrac{E_{x,1}}{f_{1,k}} & \cdots & \dfrac{\partial f_{1,k}}{\partial v_{xy,1}} \dfrac{v_{xy,1}}{f_{1,k}} & & \dfrac{\partial f_{1,k}}{\partial E_{x,n}} \dfrac{E_{x,n}}{f_{1,k}} & \cdots & \dfrac{\partial f_{1,k}}{\partial v_{xy,n}} \dfrac{v_{xy,n}}{f_{1,k}} \\ \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ \dfrac{\partial f_{m,k}}{\partial E_{x,1}} \dfrac{E_{x,1}}{f_{m,k}} & \cdots & \dfrac{\partial f_m}{\partial v_{xy,1}} \dfrac{v_{xy,1}}{f_{m,k}} & & \dfrac{\partial f_{m,k}}{\partial E_{x,n}} \dfrac{E_{x,n}}{f_{m,k}} & \cdots & \dfrac{\partial f_m}{\partial v_{xy,n}} \dfrac{v_{xy,n}}{f_{m,k}} \end{cases} \end{bmatrix} \qquad (5)$$

The performance of this new material identification routine was numerically evaluated for a coated steel plate with two different coatings on both sides of the plate. The left plot of Figure 3 gives a schematic presentation of a cross-section of the test plate, and Table 1 presents the material properties of the different layers. The top coating of sample 2 and the bottom coating of sample 3 were removed, in order to have a set of samples with different layer geometry.
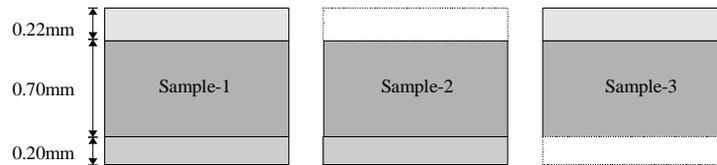

Figure 3: Geometry of the three test specimen.

Table 1: Material properties of the layers of the test specimen.

	Coating (top)	Substrate	Coating (bottom)	
E_x	61.00	201.00	71.00	GPa
E_y	60.00	200.00	70.00	GPa
G_{xy}	20.00	80.00	25.00	GPa
v_{xy}	0.180	0.200	0.190	-

Three FE-models were made to calculate the first five resonance frequencies of the sample plates 1, 2 and 3. These calculated frequencies were then inserted into the inverse procedure as the 'experimental' plate frequencies. Random values were used as starting values for the different material properties, and the correct material properties of the three layers were found after six iteration steps, Figure 4.
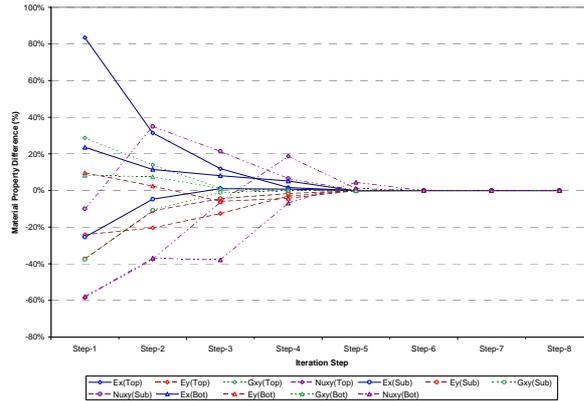
Figure 4: Convergence of the material properties.

Figure 5 shows the relation between the first column (the E_x sensitivities of the top coating) and fifth column (the E_x sensitivities of the substrate) of the global sensitivity matrix (5). It can be seen that the multi-model updating routine has succeeded in eliminating the linear relation between the columns, therefore the uniqueness of the solution of this inverse problem is guaranteed.
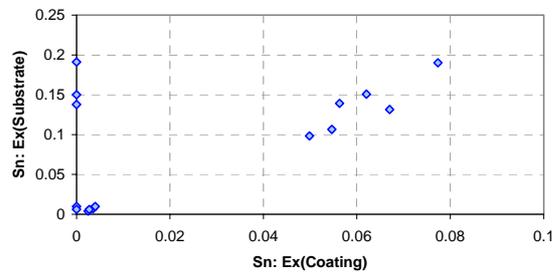

Figure 5: Relation between the first and fifth column of the sensitivity matrix.

CONCLUSIONS

This paper has described the practical implementation of a generic procedure for the simultaneous updating of multiple finite element models, each using their own set of reference test data. This combined use leads to a richer set of reference data to update parameters that are common to all FE models under consideration. The technique reduces ill-conditioning of the sensitivity matrix due to incomplete test data, and reduces the effects of inaccurate test data. The resulting updated finite element models are of better quality and more reliable. The updated parameter values are closer to a unique solution and the finite element model has a better chance to be valid outside the measured bandwidth and with different boundary conditions and loading.

The benefit of using this procedure to obtain a unique solution was demonstrated by identifying material properties of each layer of a 3-layer material (coated steel plate), using three different test specimen and three FE models.

ACKNOWLEDGEMENTS

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