Optimization of the Dynamic Response of a Complete Exhaust System

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Abstract
This paper presents an optimization approach for tailoring the dynamic response of a complete exhaust system using finite element modeling. Before the optimization procedure is started, the FE-model is updated using modal test data. This preliminary step is required to ensure the validity of the initial FE-model. The actual optimization routine consists of two iteration loops: an inner and an outer loop. The inner iteration loop performs the optimization using a modal domain modification technique to predict the change of the dynamic response of the structure. In this way the structure can be optimized in a computational efficient way. However, the modal domain prediction is only accurate within a limited parameter range. Therefore, the outer iteration loop re-evaluates the full finite element model once the parameter changes exceed the 'trust-region' bounds of the modal domain prediction. The solution of the re-evaluation is then used as improved base for the modal domain prediction in the inner iteration loop.
The suggested optimization approach is illustrated on a finite element model of an exhaust system of a passenger car. The exhaust system is connected to the car body using four isolators. The optimization is performed to keep the force transmitted by the exhaust system through the isolators to the car body below the design specifications, optimizing the stiffness of the decoupling elements. The goal is to ensure a good NVH (Noise Vibration Harshness) performance of the exhaust system.

Nomenclature

$[\bullet_m]$	Modal matrix
$[\bullet_e]$	Element matrix
Ψ	Mass normalized mode shape
B	Damping matrix (viscous damping)
C	Damping matrix (structural damping)
$HR(\omega)$	Harmonic function
K	Stiffness matrix
M	Mass matrix
p_i	Optimization or model parameter

1. Introduction

Automotive industry requirements for quality, productivity and cost efficiency are at a level where optimization of the design and manufacturing of a product must occur in the earliest stages of conception. The finite element (FE) optimization approach allows an efficient evaluation of designs using advanced mathematical tools. However, methods for the practical application of the FE optimization approach must still be developed.
Exhaust systems present a special case because of their geometry and the constraints placed on their design by the underbody of the car. Exhaust systems are submitted to many dynamic input loads, the most important one coming from the engine. The induced vibrations are spread along the exhaust system, and forces are transmitted to the car body through the attached points. To reduce the force levels, decoupling elements like ball-joints or

flex-couplings, and isolators are used. As these vibrations could induce structural borne noise in the passenger compartment, it is necessary to define the best decoupling elements that allow reduce their amplitude to meet their target values. The objective of this study was to develop and verify a procedure for the optimization of the dynamic response of a complete exhaust system. Note that this procedure includes a correlation and model-updating step, in order to ensure the validity of the FE-model that is used in the optimization procedure.

2. The Exhaust System

2.1. System Description

An exhaust system is made up of pipes, acoustic and emission control components and decoupling elements. The exhaust system used in this paper has a ball-joint at the inlet pipe, two acoustic volumes and is hanged using four isolators. The front ones and the rear ones are similar in pairs. The FE-model of the considered exhaust system consists of 107156 elements (mainly quad4) and has 646200 degrees of freedom.

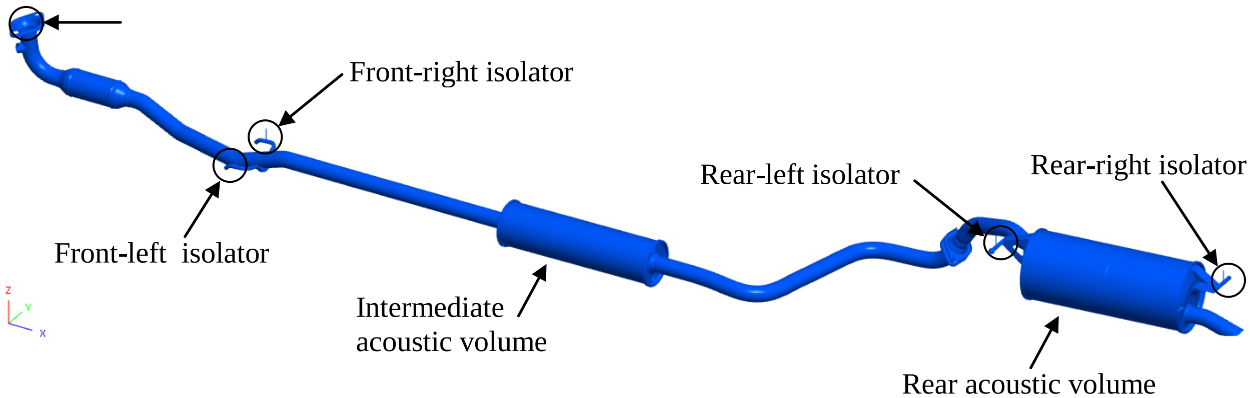

Figure 1: Exhaust system description.

2.2 Model Updating

Before the exhaust system is optimized, the validity of the initial FE-model is verified using a correlation analysis and is improved by updating the FE-model where necessary. Modal data, resonance frequencies and mode shapes, are used as reference data.

2.2.1. Test Set-up and Results

Two different modal tests were performed. In the first modal test, the exhaust system was suspended with free-free boundary conditions. During the second modal test, the exhaust system was installed on a dynamic test bench in a similar way as it would be in operational conditions, this test set-up is shown in the figure 2.

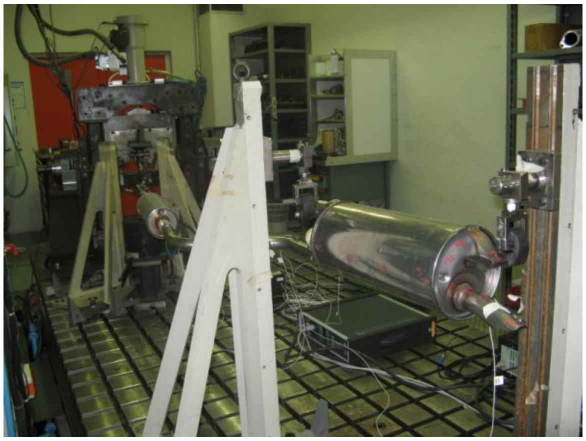

Figure 2: Exhaust system fixed on the test bench.

The locations of the measurement and excitation points were defined directly on the exhaust system using a 3D metrology tool.

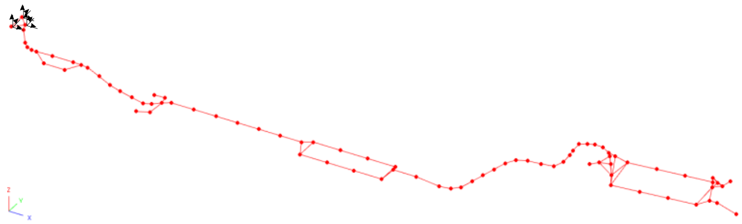

Figure 3: The measurement and excitation points.

The acquisition of experimental data was carried out using shaker excitation. Tri-axial accelerometers were used to measure the response in every point. The experimental modal basis was extracted from the FRF measurements using conventional modal parameter estimation methods.

2.2.2. Updating

Two model updating procedures were performed using the FEMtools Model-Updating module [1]. The first one used the modal test data obtained under free-free boundary conditions. The goal of this updating procedure was to improve the reliability of the FE-model of the exhaust system (without the decoupling elements). The second updating procedure was performed with the modal test data obtained using the operational boundary conditions. Before this second updating procedure was started, the ball-joint and isolators were added to the FE-model that resulted from the first updating procedure. For the second updating, only the stiffness of the decoupling elements were considered as updating parameters. The goal of the second updating procedure was to verify whether the FE-model of the exhaust system was reliable enough to reproduce the test data obtained under operational conditions. As the only uncertainty was in the decoupling elements, only the stiffness of these elements was modified. Table 1 show the correlation between the FEM and EMA data after the second model-updating step

Pair	FEA [Hz]	EMA [Hz]	Diff. [%]	MAC [%]	Pair	FEA [Hz]	EMA [Hz]	Diff. [%]	MAC [%]
1	11.59	11.60	-0.13	96.8	11	130.64	129.24	1.08	97.2
2	18.08	18.14	-0.35	93.7	12	152.46	148.33	2.78	96.0
3	21.54	22.14	-2.72	97.7	13	184.25	181.88	1.30	96.4
4	25.13	25.53	-1.56	94.9	14	223.44	218.71	2.16	95.3
5	42.69	42.52	0.41	98.4	15	277.47	266.71	4.04	80.3
6	44.66	45.50	-1.85	93.6	16	291.62	283.25	2.96	93.6
7	66.28	65.85	0.65	97.8	17	320.93	312.23	2.78	89.8
8	77.11	75.77	1.78	96.8	18	341.87	335.11	2.02	82.4
9	94.54	97.40	-2.93	94.9	19	362.31	360.42	0.52	86.8
10	115.55	114.28	1.11	97.4					

Table 1: Analytical and experimental modal basis correlation after updating.

3. Response Computation

3.1. Harmonic Response Computation

The goal of the harmonic response computation is to compute the force transmitted through the isolators to the car body as a function of frequency. The computation of the responses using a direct method is too slow to be used for optimization purposes. Therefore, the harmonic responses are computed using a modal-based approach [2-3].

The engine behavior is mainly defined by the displacement of the piston along its axis. For that reason, only this component was taken into account to compute the exhaust system response. In the considered case study, acceleration measurements were carried out on the exhaust system fixed on a car in run up conditions. The acceleration data at the inlet of the exhaust system were used as input load for the dynamic simulations.

The amplitudes value of the hanging forces are very sensitive to the damping assumptions. For this reason experimental modal damping values were used to model the damping of the exhaust system; structural damping was used to model the damping in the hangers and ball-joint. The vertical hanging force values computed under the above assumptions are given in figure 4. Note that the forces transmitted in the *Z*-direction exceed the acceptance criterion level, and this for all four isolators.

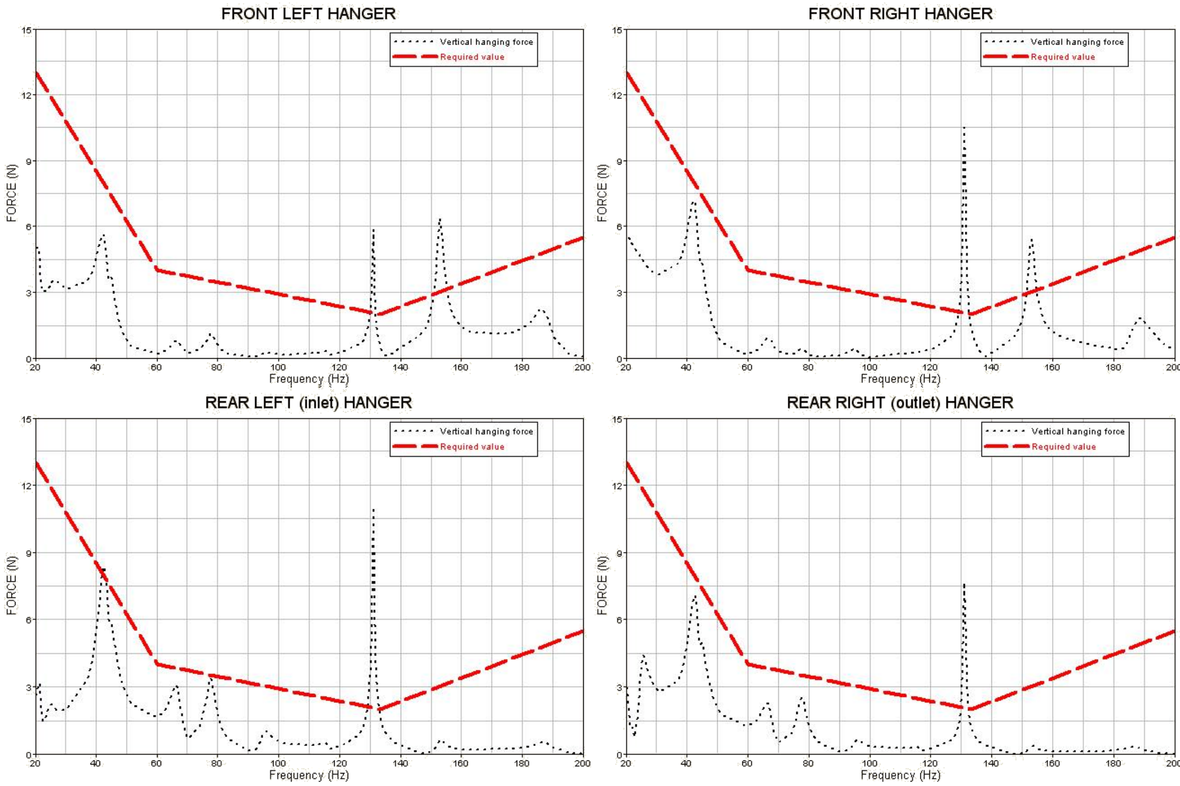

Figure 4: Hanging forces under engine load. The transmitted forces (black) compared to the design specifications (red) in the local *Z*-direction.

3.2. Harmonic Response Modification

The harmonic responses of the modified model can be obtained by computing the resonant frequencies and modes shapes of the modified structure, and using these new frequencies and mode shapes to compute the transmitted forces. The main disadvantage of this approach is that every modification requires an additional run of the FE-solver to compute the frequencies and mode shapes.

The modal-based harmonic response computation allows an alternative approach to estimate the impact of a modification of the system on the harmonic responses. Instead of re-computing the mode shape vectors, the initial vectors are used to compute the effect of the modification on the model stiffness, damping and mass matrices.

$$\begin{gathered}[\Delta K_m]=\sum_{i=1}^{n_p}\left([\Psi_e]^T\left([K_e(p_i+\Delta p_i)]-[K_e(p_i)]\right)[\Psi_e]\right)\\ \vdots\\ [\Delta M_m]=\sum_{i=1}^{n_p}\left([\Psi_e]^T\left([M_e(p_i+\Delta p_i)]-[M_e(p_i)]\right)[\Psi_e]\right)\end{gathered} \tag{1}$$

in which n_p is the number of modified model parameters, p_i is the initial value of the i^{th} parameter, Δp_i is the modification of the i^{th} parameter, $[\Psi_e]$ is the shape vector matrix reduced to the DOFs of element e, and $[K_e]$ is the element stiffness matrix.

Based on the results of expression (1), the modal matrices of the modified system can be estimated from the modal matrices of the initial system.

$$\begin{aligned}[K_m^{\mathrm{mod}}]&=[K_m]+[\Delta K_m]\\ [B_m^{\mathrm{mod}}]&=[B_m]+[\Delta B_m]\\ [C_m^{\mathrm{mod}}]&=[C_m]+[\Delta C_m]\\ [M_m^{\mathrm{mod}}]&=[M_m]+[\Delta M_m]\end{aligned} \tag{2}$$

The transmitted forces of the modified system can now be approximated using these modified modal matrices for the harmonic response computation. The modal-based prediction of the effect of a modification of a model parameter on the transmitted forces appears to be very reliable, even for relatively large parameter modifications. Consider a reduction of the vertical stiffness of the front-left isolator with 25%. When comparing the transmitted force obtained using re-analysis with the transmitted force obtained using the modal-based prediction, the maximal offset between the amplitude was less than 1%.

4. Optimization of the Complete Exhaust System

4.1. Goal

The goal of the optimization is to bring all the forces in the isolators below the acceptable level, by modifying the structure as little as possible. In this step of the development, the geometry of the exhaust system is frozen (size and location of the volumes, layout and hanging points cannot be changed). Changes to the decoupling elements are the only changes that can be considered. The target is now to define the optimal stiffness values of ball-joints and isolators that will allow reaching the design requirements.

4.2. **Parameter Impact Study**

There are 18 potential optimization parameters in the FE-model: 3 translational and 3 rotational stiffness coefficients of the ball-joint (6), and the 3 translational stiffness coefficients of the 4 isolators (12). It is, however, not certain that all those parameters have a significant impact on the considered forces.

The effect of these potential optimization parameters on the transmitted forces was evaluated by modifying the parameters one by one, using a parameter modification of 25%, and verifying the correlation between the harmonic responses of the initial and perturbed model. The correlations between the harmonic responses are quantified by the signature assurance criterion (SAC) and amplitude difference (AD) values between the amplitudes of the two considered response functions. The SAC and AD value between two frequency functions are defined as [4]:

$$SAC = \frac{\left| HR(\omega)_1^T \; HR(\omega)_2 \right|^2}{\left(HR(\omega)_1^T \; HR(\omega)_1 \right)\left(HR(\omega)_2^T \; HR(\omega)_2 \right)} \tag{1}$$

$$AD = \frac{\sum_{\omega} HR(\omega)_2 - \sum_{\omega} HR(\omega)_1}{\sum_{\omega} HR(\omega)_1} \tag{2}$$

The SAC value describes the difference in shape between two responses, while the AD describes the difference in the average amplitude between two compared functions. The SAC and AD values were computed for the transmitted forces in the Z-direction of the four isolators, and this for the 18 potential optimization parameters. The impact study showed that the effect of a modification of the three translational stiffness coefficients of the ball-joint on the shape and the amplitude of the considered response functions are insignificant. The effect of the rotational stiffness coefficient H_x of the ball-joint is insignificant as well. The stiffness coefficients of the isolators have both an effect on the shape and on the average amplitude of the transmitted forces. However, a modification of the stiffness in a particular direction of an isolator mainly affects the force transmitted in the considered direction of the considered isolator. The effect on the forces transmitted in the other directions, and through the other isolators is limited. Furthermore, to reduce the number of different parts in the final design, the stiffness of the two isolators in the front, and the isolators in the rear are requested to be identical. In the end, only four optimization parameters are retained, table 2 gives an overview.

Parameter	Description
H_y	Rotational stiffness of the ball-joint around the local Y-axis
H_z	Rotational stiffness of the ball-joint around the local Z-axis
D_1	Scaling factor of the stiffness of the front isolators
D_2	Scaling factor of the stiffness of the rear isolators

Table 2: The selected optimization parameters.

4.3. Optimization

The objective function of the optimization problem is defined as the sum of squares of the relative changes of the optimization parameters. The use of this objective function thus results in an optimized model that differs as little as possible from the initial model. The acceptable levels for the forces transmitted through the isolators are added as constraints. Mathematically, the optimization problem is expressed as:

$$
\begin{array}{lll}
\text{Minimize} & f_0 = \sum_{i=1}^{6} \left(\dfrac{p_i^0 - p_i}{p_i^0} \right)^2 & \\
\text{Subject to} & g_i = F_i(\omega) - F_a(\omega) \le 0 & \forall i = 1,\ldots 12 \\
 & p_i \ge p_i^{\min} & \forall i = 1,\ldots 4 \\
 & p_i \le p_i^{\max} & \forall i = 1,\ldots 4
\end{array}
\tag{3}
$$

in which p_i^0 and p_i are the reference[1] and current values of the optimization parameter, $F_i(\omega)$ is the transmitted force of the i^{th} response, and $F_a(\omega)$ is the admissible force level as a function of frequency. The side constraints on the optimization parameters represent the feasible range of stiffness values for the ball-joint and isolators.

The optimization is performed with the FEMtools Optimization module using a gradient-based optimizer [5], which requires the gradients of the objective and constraint functions. The evaluation of the gradients of the objective function is straightforward. The gradients of the constraint functions can be computed with a finite difference approach using the harmonic response modification that was introduced in section 3.2.

The optimization is performed using a double iteration loop. The inner iteration loop optimizes the values of the optimization parameters using only the modal-based harmonic response modification technique. The mode shapes are never recomputed in the inner iteration loop. Once the solution in the inner iteration loop has converged, the inner loop is aborted, and the optimization procedure returns to the outer iteration loop. The outer iteration loop just re-computes the mode shapes based on the current parameter values. Once the new shapes are computed, a new inner iteration loop is started. In this way, the number of (computationally expensive) mode shape evaluations can be minimized. The overall solution was considered to be converged when the inner iteration loop modified the parameters less than 0.25%. The whole procedure was automated using the FEMtools Script programming language.

The optimization problem was solved using the updated values of H_y and H_z, and the minimal allowed values for K_z. These values were chosen because they provide a feasible design. The values of the parameters were then optimized in order to obtain a model that was as close as possible to the updated model, while respecting the imposed limit on the transmitted forces. Table 3 presents the results obtained.

Parameter	Modification with respect to the updated model
H_y	+ 0.0%
H_z	+ 0.0%
D_1	- 83.0%
D_2	- 82.7%

Table 3: The optimal design.

The optimization routine did not modify the stiffness of the ball-joint. It thus appears that the cost (deviation from the initial design) for modifying the stiffness of the ball-joint is higher than the benefit (the reduction of the transmitted force). The stiffness of both the front and rear isolators had to be reduced by 83% in order the meet the design specifications. Figure 5 compares the transmitted force spectra of the initial and optimized FE-model with the design specification limits.

Table 4 provides an overview of the iteration histories of the optimization procedure. The whole optimization procedure took about 13 minutes[2] to solve. Note that without the use of the modal-based approximation technique it would take about 2 hours.

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[1] The values of the updated FE-model.

[2] All calculations were performed on a standard PC with an Intel dual-core 6600 2.4GHz processor and 4 GB of RAM running Windows Vista 64-bit.

Step	Parameter				Iterations	Time [s]	
	H_y	H_z	D_1	D_2			
Start	0.0%	0.0%	84.0%	92.8%			208.9
1	0.0%	0.0%	82.6%	82.4%	5	232.7	209.5
2	0.0%	0.0%	83.0%	82.7%	2	130.6	
Result	0.0%	0.0%	83.0%	82.7%	Total		781.7

Table 4: The iteration histories.

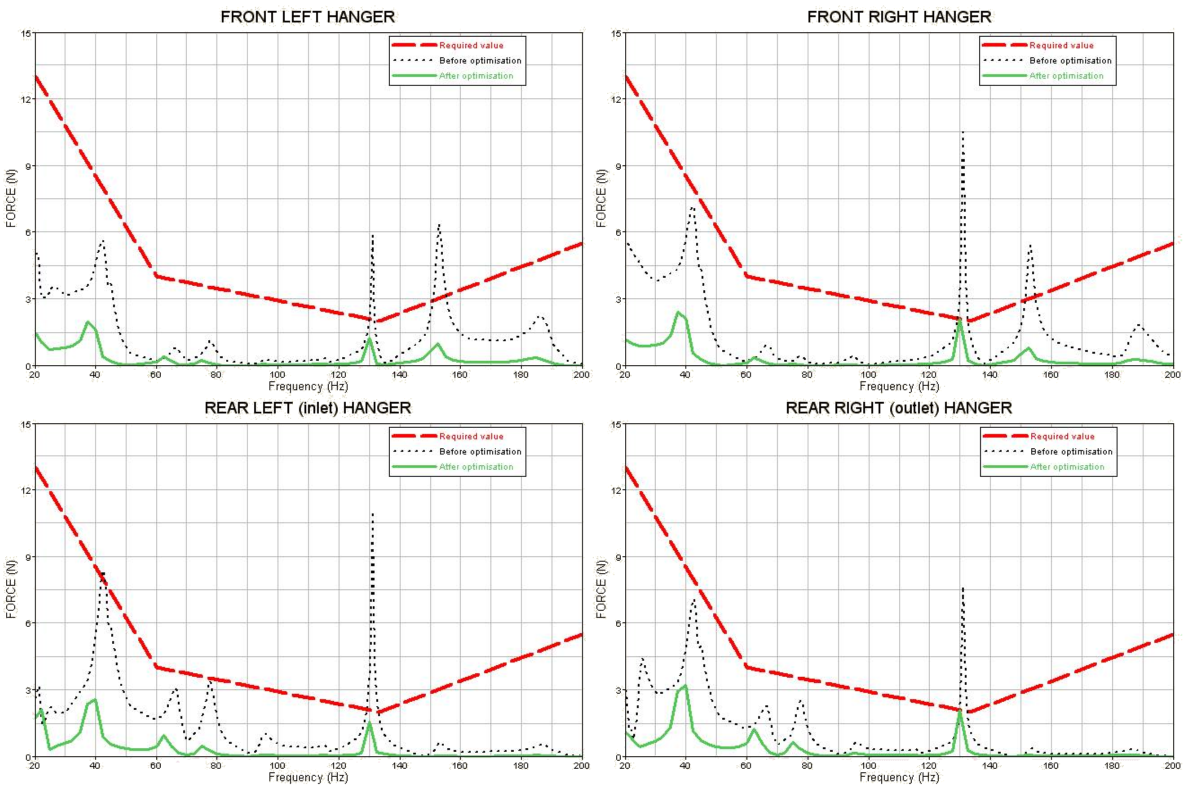

Figure 5: Hanging forces of the optimized model under engine load. The transmitted forces (black) compared to the design specifications (red) in the local Z-direction.

5. Conclusions

This article presented an optimization approach to optimize the harmonic response of industrial-sized FE-models. The presented optimization approach was successfully used to optimize the dynamic performance of a complete exhaust system. The following conclusions can be drawn for the presented case:

1) The modal-based approximation technique appears to be well suited to solve optimization problems like the one presented in this paper. For the considered exhaust system, the use of the modal based-approximation technique reduced the required computation time with a factor of about 10. The benefit of using the modal-based approach will further increase with an increasing number of optimization parameters and model size.
2) The predictive range of the modal-based approximation technique appeared to be larger than initially expected. Note that the parameters that were selected in the presented test case had a relatively limited effect on the mode shapes. As the modal-based approximation heavily relies on the mode shapes, its predictive capability might go down if the selected optimization parameters have a severe impact on the mode shapes.
3) Until now, the use of mathematical tools allowed to study the dynamic behavior of full exhaust systems under engine loads in early stages of conception. With the methodology presented in this paper, it is possible to go forward, optimizing components from the beginning of the development process. The time

spent in trial and error analysis to evaluate the impact of the characteristics of the decoupling elements on the exhaust system response is saved. With the presented approach, the process of finding the best compromise between changing the stiffness of the ball-joint and the isolators is automated. It is now possible to identify the most important parameters, and to meet the customer's requirement using the FE model of the complete exhaust system.

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