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Modal Testing and FE Model Updating of a Lively Staircase Structure

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ABSTRACT

Effects of structural flexibility on the dynamic performance of structures such as staircases, footbridges, and long span floors is becoming an increasingly important aspect of modern design. Cost reduction, improving efficiency of design, enhancement of aesthetic perception and, innovation in architectural forms often result in slender and lightweight structures that are significantly more flexible and vibration-prone than ever before. Consequently, meeting relevant vibration serviceability criteria, as opposed to ultimate strength requirements, is becoming the governing factor in the design of many new structures. Despite significant advances in numerical prediction of modal properties of structures using Finite Element (FE) modelling technique, there still exist challenges in accurate representation of the actual dynamic behaviour. This is mainly due to some inherent modelling uncertainties related to a lack of information on the as-built structures, such as uncertainties in boundary conditions, material properties and the effects of non-structural elements. This paper presents the results of a modal testing exercise carried out to assess the dynamic behaviour of a lively staircase structure. The assessment procedure includes a full-scale ambient vibration testing, modal identification and FE modelling and updating. In particular, the influence of boundary conditions and presence of handrails on dynamic properties of the structure are commented.

1. Introduction

In recent decades, it has become increasingly popular to provide educational buildings, hotels, hospitals and other public areas with slender and lightweight staircases, often for aesthetic reasons. One inherent characteristic of this type of design is a low stiffness to mass ratio typically producing lower natural frequencies when compared to more traditional staircase designs. As a result, many staircases are dynamically responsive and the vibration serviceability criteria are becoming the governing factors in the design of this kind of structures. In the case of low-frequency and lightly damped staircases, their dynamic response due to near-resonant excitation governs their vibration performance and simulations of this type of near-resonant dynamic response is very sensitive to even small variations in modal properties. Therefore, knowing modal properties of the staircase and its mode shapes as precisely as possible is very important not only for the design of new structures with similar layouts, but also for the rectification of existing lively staircases. However, despite significant advances in numerical prediction of modal properties of structures using FE models, there still exist challenges in accurate representation of the actual dynamic behaviour. The main reason for this is the general lack of information on the as-built structures, such as uncertainties in boundary conditions, material properties and the effects of non-structural elements.

A possible approach for filling the gap between the real structural performance and the FE models is to employ some form of modal testing [1]. The key idea is that the FE model can be verified and improved by correlating the natural frequencies and mode shapes estimated from the model with those obtained from the modal testing. Although the modal testing is one of the most popular techniques for studying the dynamic behaviour of engineering structures, there are only a few articles related to both experimental measurement and FE modelling staircases [2,3].

Once the modal properties of the staircase (mainly natural frequencies and mode shapes) are identified experimentally, the level of error introduced by the initially developed FE model can be identified together with the drawbacks in the FE modelling and the initial FE model can be corrected by means of FE model updating techniques [4,5].

Bearing all this in mind, this paper presents a case study related to FE modelling, modal testing and FE model updating of a lively staircase. It demonstrates shortcomings of a detailed FE model, which was firstly developed by employing the best engineering judgment and available design data. The lowest modes of vibration in the vertical direction were identified using a frequency-domain parameter estimation technique. Based on the experimental results, the initial FE model was revised and manually tuned to match frequencies of the staircase more closely. Manual tuning is a necessary condition for preparation for the automatic updating procedure [6] that is conducted as a final part of this study.

2. Description of the test staircase structure

The selected staircase structure was constructed in 2004 and is located in the Zeeman building on the University of Warwick Campus (Figure 1). The staircase is attached to a concrete upper floor and a lower bridge-like structure supported by concrete columns. Around the midspan, the structure is also connected on one side to a side wall by means of two screws (Figure 2). This lively structure is made of steel and concrete. It consists of nineteen steps and a midspan landing. The steps and the landing are supported on two 20 mm thick steel plates. Each step is made of a steel U-shaped base with a mortar infill of 60 mm thickness. The steps are bolted in-between the two side plates. The landing is stiffened by a 120x60x5RHS steel hollow beam framework bolted to the side plates. In addition to the steel plates, the bottom of the stairs is supported by a 120x60x5RHS while the top is supported by a 20 mm thick steel plate. A handrail attached to a steel plate runs along the whole length of the staircase. More information about the geometry of the staircase structure is given in Figure 3.

Figure 1. Zeeman building and staircase structure

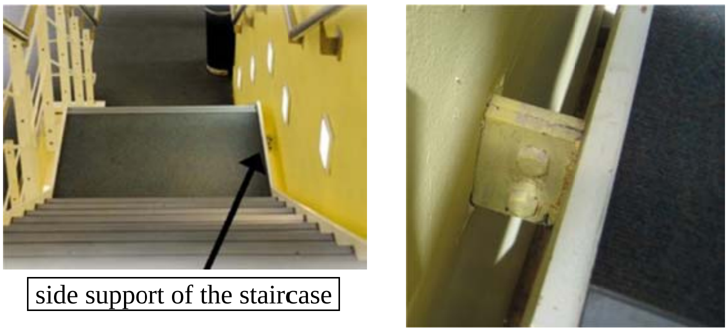


Figure 2. Detail of the side support of the staircase

One uncertainty in the definition of the FE model of the staircase was the material properties of the mortar. The assumed values of the materials properties of the mortar used in the initial analysis of the staircase, together with the material properties of the steel, are listed in Table 1. The mass of the staircase structure was estimated to be 2.20 tonnes.

	Young's modulus [N/m^2]	Density [kg/m^3]	Poisson's ratio
Steel	2.00E+11	7860	0.3
Mortar	2.50E+10	2400	0.2

Table 1. Material properties

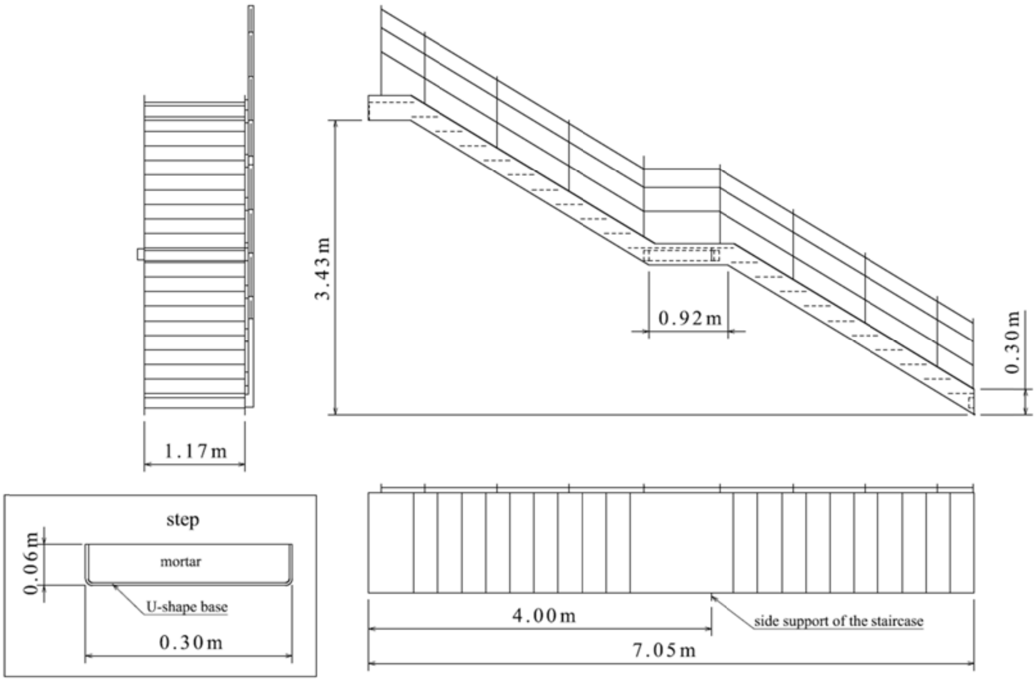


Figure 3. Geometry of the staircase

3. Initial FE modelling

A good practice for modal testing of an as-built structure requires a development of reasonably detailed FE model before the testing. This first insight into dynamic behaviour of the staircase helps the test planning and preparation.

A three dimensional (3D) linear elastic FE model for the staircase structure (Figure 4) was developed using software ABAQUS [7]. The aim was to develop a detailed initial model which would be able to simulate the dynamic behaviour of the staircase as well as possible. This was based on the limited technical data available and the best engineering judgement. The key modelling assumptions were as follows:

- Each part of the staircase structure, except the bars of the handrail, was modelled using orthotropic shell elements (QUAD4).
- The bars of the handrail were modelled using beam elements (BEAM2).
- Side support and supports at both ends of the staircase were modelled as pinned.

The first two bending modes and the first torsional mode of vibration obtained from the initial FE model are shown in Figure 5. Labels B and T stand for the bending and torsional modes, respectively.

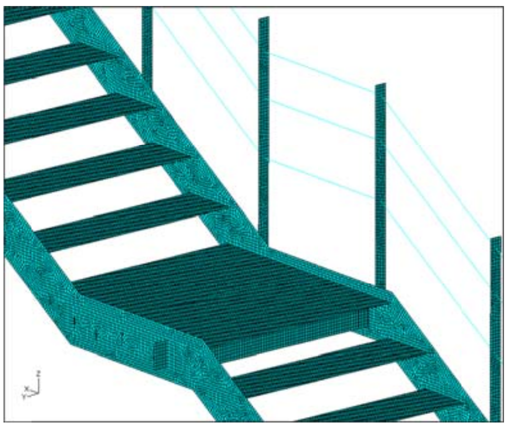
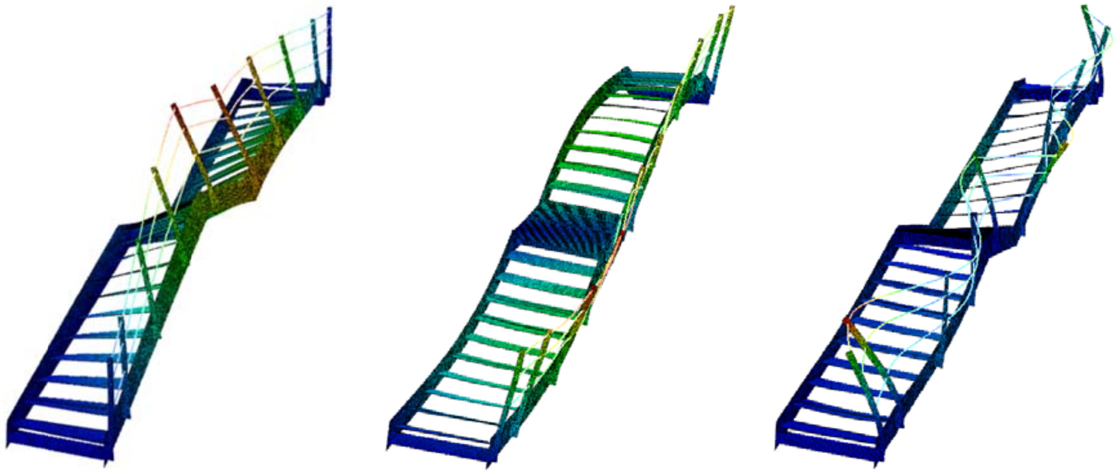


Figure 4. Detail of the FE model of the staircase



1B: 11.39 Hz 2B: 20.58 Hz 1T: 39.57 Hz

Figure 5. Modes of vibration calculated from the initial FE model

4. Ambient vibration testing

An Ambient Vibration Survey (AVS) was carried out to identify the modal properties of the staircase structure taking into account the information obtained from the initial FE model. Only vertical response measurements due to ambient excitation on the empty staircase were made.

The equipment used for the measurements includes 4 accelerometers QA750 (Figure 6a) and 4-channel NI-9234 data acquisition card. To identify the mode shapes and natural frequencies of the staircase and the interaction between the staircase and the adjacent elements, response measurements were made on the staircase, as well as in surrounding areas: floor at the top of stairs, bridge at the bottom and the bridge parallel with the stairs. Accelerometer locations are shown in Figure 6b. The reference accelerometer was located at the mid-landing, pointed out in dark colour in Figure 6b, while the traveller accelerometers were placed at different points to complete 23 setups.

(a) (b)

Figure 6. Accelerometer locations

The data acquisition parameters adopted for the AVS were 200 s acquisition time for each setup, recorded at a time step of 6.05E-4 s. Typical measured time history at the reference point is shown in Figure 7.

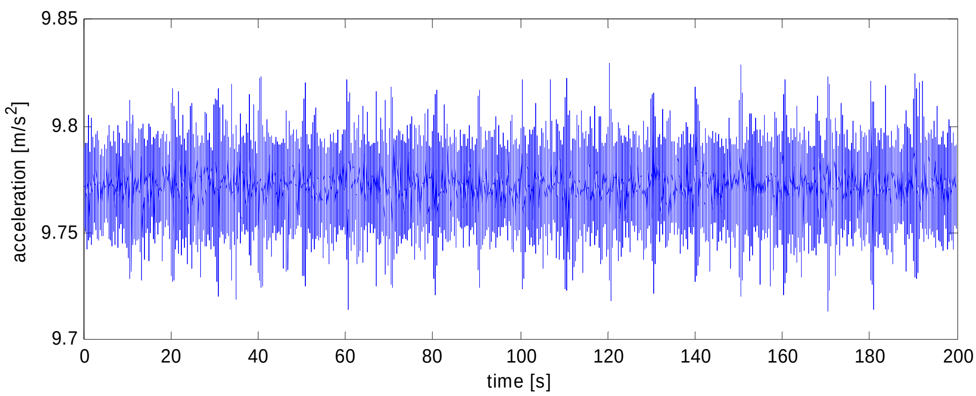


Figure 7. Acceleration time history at the reference point

5. Modal parameter identification

For modal parameter identification using the AVS data, ARTeMIS software [8] was used and the Frequency Domain Decomposition (FDD) technique was chosen. The FDD is an extension of the Basic Frequency Domain (BFD) technique, or more often called the Peak-Picking technique. This approach is based on the idea that modes can be estimated from the spectral densities calculated, assuming a white noise input, and a lightly damped structure. It is a non-parametric technique that estimates the modal parameters directly from signal processing calculations [9]. The FDD technique estimates the modes using a Singular Value Decomposition (SVD) of each of the data sets. This decomposition corresponds to a Single Degree of Freedom (SDOF) identification of the system for each singular value.

Results of FDD peak picking method from all the measurement setups are presented in Figure 8. Only those peaks corresponding to the vibration modes with dominant vibration of the staircase were identified, while modes with dominant vibration of the surrounding structure are not presented in this paper. The identified mode shapes and natural frequencies are shown in Figure 9.

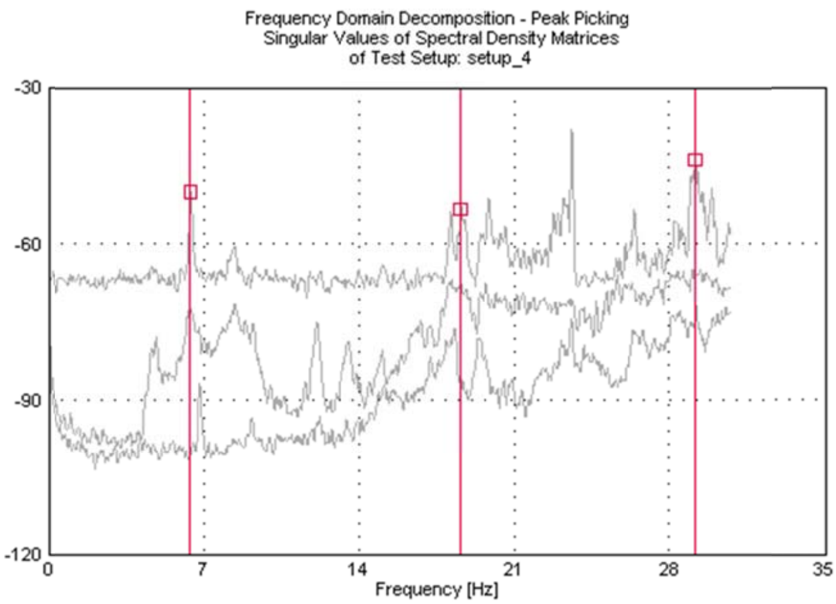
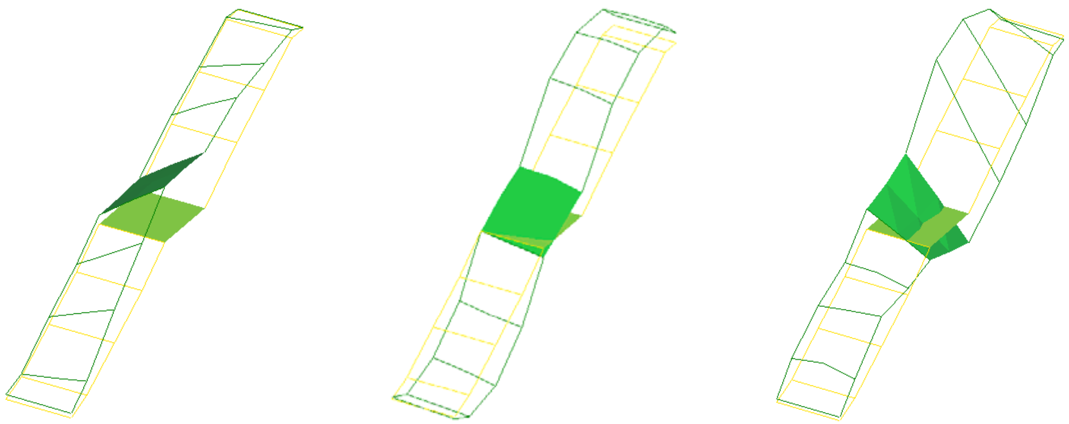


Figure 8. Modes identified in AVS



1B: 6.37 Hz 2B: 18.63 Hz 1T: 29.03 Hz

Figure 9. Mode shapes and natural frequencies obtained experimentally

From some preliminary studies, it is known that the first vibration mode at around 6.3 Hz is responsible for liveliness of the staircase. To study this mode in more detail, a free vibration test was conducted. A person was jumping on the staircase at 3.15 Hz and then walked off the structure. The free decay response recorded is shown in Figure 10. The natural frequency and the damping ratio were then identified from ten successive cycles of the free decay using the logarithmic decrement method and they are shown in Figure 11 and Figure 12, respectively. The average damping ratio is 0.6% for vibration amplitudes above 1.0 m/s^2, 0.54% for vibration between 0.3–1.0 m/s^2, about 0.47% for vibration 0.1–0.3 m/s^2 and 0.41% for vibration below 0.1 m/s^2.

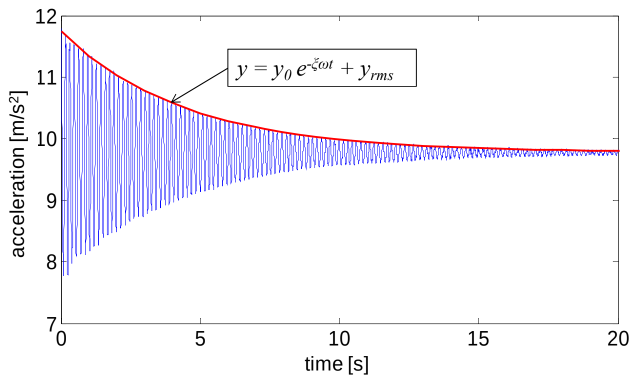


Figure 10. Recorded free decay and free decay response envelope

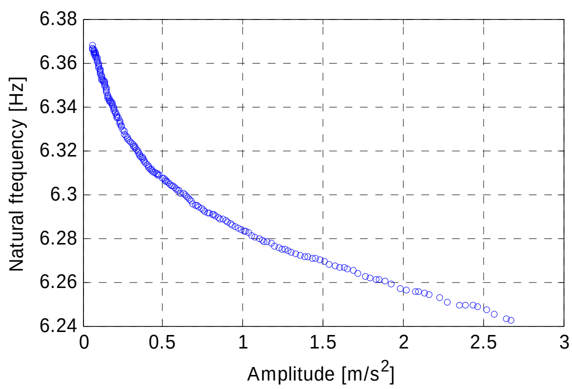


Figure 11. Natural frequency vs. amplitude

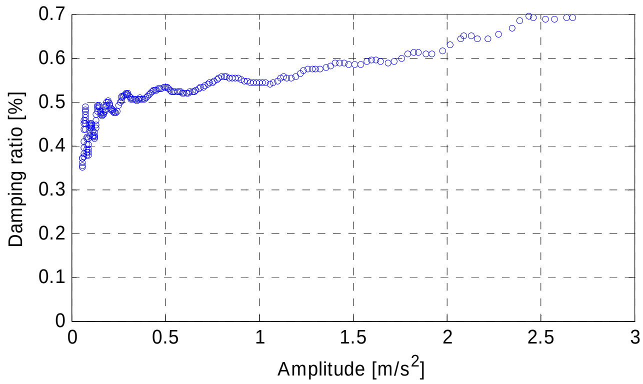


Figure 12. Damping ratio vs. amplitude

6. FE model tuning

A comparison between vibration modes obtained experimentally and numerically using the initial FE model is shown in Table 2. Natural frequencies of all experimental modes were overestimated, with the frequency error being exceptionally high for mode 1B (78.81%) and mode 1T (36.31%). On the other hand, the correlation between mode shapes was good with Modal Assurance Criterion (MAC) values higher than 95%, except for mode 1T (70.4%). It can therefore be concluded that there was a problem with inadequate modelling which overestimated the stiffness of the staircase.

Mode number	Modal testing f_I [Hz]	Initial FE model f_{II} [Hz]	Difference $(f_{II} - f_I)/f_I$ [%]	Mode shape correlation MAC [%]
1B	6.37	11.39	78.81	96.2
2B	18.63	20.58	10.47	95.4
1T	29.03	39.57	36.31	70.4

Table 2.Correlation between experimental and initial FE model

Manual tuning was required to get a FE model of the staircase suitable for automatic updating procedure. Pinned boundary condition in y- and z-direction at the side support and in x- and z-direction at both ends of the staircase was removed and springs were introduced in the corresponding directions in the FE model. So, as it is shown in Figure 13, K_{xi}, K_{zi}, i = 1,..,4 were placed at both ends and K_{y1}, K_{y2}, K_{z5} at the side support of the staircase.

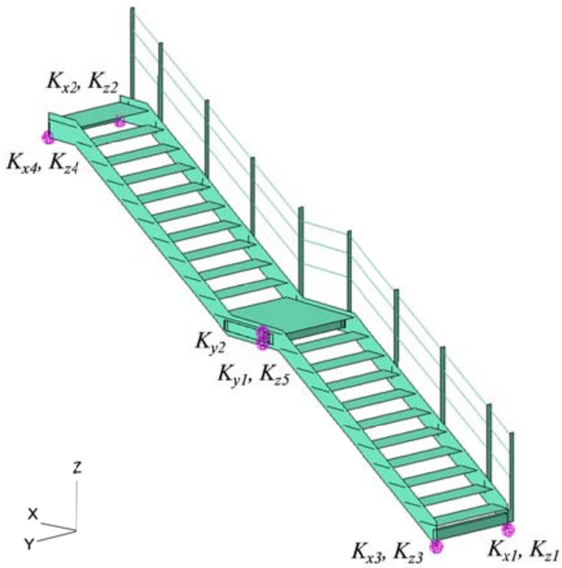


Figure 13. Springs added to the tuned FE model

The stiffness of these springs was varied by trial and error until the best correlation with measured frequencies was obtained. A stiffness value of 500 kN/m for lateral support and 10000 kN/m for supports at both ends of the staircase produced the smallest difference between the measured and natural frequencies calculated from the FE model for the mode shapes considered. These values were adopted in the manually tuned FE model developed prior to automatic updating. Frequency error decreased significantly with the maximum value being 13.03% for mode 1B (Table 3). The MAC values changed only slightly.

Mode number	Modal testing f_I [Hz]	Tuned FE model f_{II} [Hz]	Difference $(f_{II} - f_I)/f_I$ [%]	Mode shape correlation MAC [%]
1B	6.37	7.20	13.03	99.1
2B	18.63	18.62	-0.04	94.5
1T	29.03	29.72	2.38	71.5

Table 3. Correlation between experimental and manually tuned FE model

7. Automatic model updating

The updating procedure was conducted in order to improve further the analytical model so it could be used in more advanced vibration response analysis, which is beyond the scope of this paper.

7.1 Target response selection

The three measured modes of vibration (1B, 2B and 1T) were targeted in the updating process and the measured natural frequencies were taken into account. Therefore, in total three target responses were selected for updating.

7.2 Parameter selection

The main criteria for parameter selection were their uncertainty and sensitivity. Therefore, parameters related to the material properties of the mortar and the thickness of the steps were selected as uncertain. This is because of uncertain contribution of the composite steps to the stiffness of the staircase. Finally, the stiffness of the springs introduced in the manually tuned FE model were also taken into account. In total, 14 parameters were selected for the updating process with their starting values given in Table 4.

7.3 Updating and results

The updating procedure was conducted using the FEMtools updating software [10] based on the Bayesian technique. The Bayesian parameter estimation expression includes the use of weighting coefficients on the parameters as well as on the responses. The aim was to minimise the error function which includes differences, not only between the target experimental and numerical responses, but also between updating parameters in two successive iterations as well as parameters and target responses' weights. Upper and lower allowable limits for parameter values were introduced in the updating procedure (Table 4). The parameter changes per iteration were not limited.

The updating process converged after six iterations. The natural frequencies and MAC values obtained as a result of the updating are presented in Table 5. It can be seen that previous maximum frequency difference 13.03% decreased to 0.1%. Minimum MAC value increased from 71.5% to 82.1%, with other two values being well above 90%. The agreement between mode shapes in updated FE model and the experimental data was good, which can be seen in Figure 14.

The final parameter values are presented in Table 5. The absolute maximum parameter change was 41.05% for the stiffness of a support spring. It should be said that an attempt to update the initial (i.e. not manually tuned) FE model led to much worse frequency and MAC correlation. Therefore, the manual model tuning conducted before the FE updating proved to be crucial for the success of the updating procedure. The mass of structure obtained as a result of the updating process using the FEMtools updating software was 2.19 tonnes, which is very close to the initially estimated mass (2.20 tonnes).

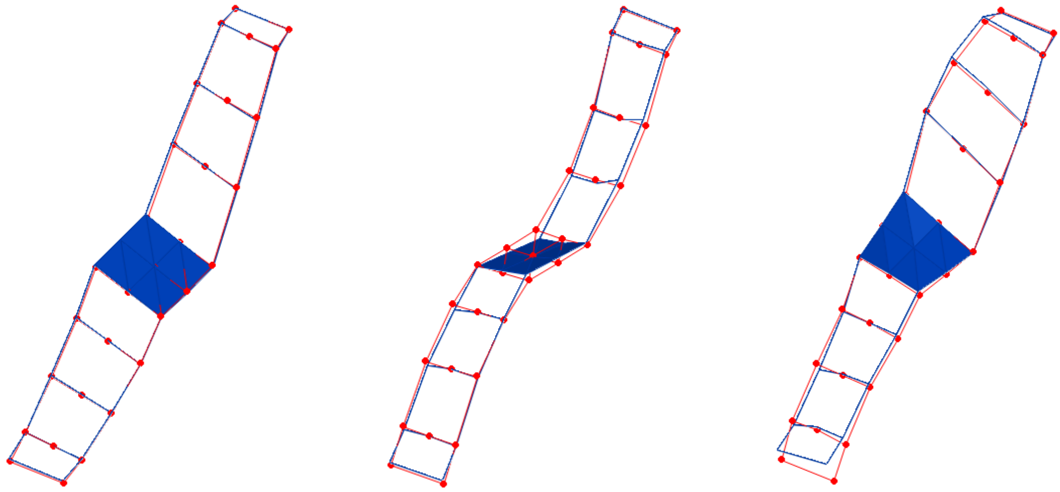
Finally, having in mind that the first bending mode of vibration is responsible for the staircase liveliness and important for further vibration analysis, the modal parameters related to this mode were possible to be identified by combining the FE and experimental results. These are an amplitude dependent natural frequency varying between 6.24 Hz and 6.37 Hz (from testing), damping ratio, which is also amplitude dependent and varies between 0.41% and 0.6% (from testing) and modal mass of 683 kg (from updated FE model). It should be pointed out that the computed value of the modal mass is expected to underestimate the actual modal mass, bearing in mind that the staircase moves together with the surrounding structure to which the staircase is attached, so the first bending mode involves more mass than it was modelled in the FE model.

Parameter number	Type	Structural part	Allowed decrease [%]	Allowed increase [%]	Starting parameter value	Updated parameter value	Parameter change [%]
1	E	steps	-20	20	2.50E+4 (MPa)	2.09E+04 (MPa)	-16.32
2	ρ	steps	-10	10	2.40E+3 (Kg/m^3)	2.50E+03 (Kg/m^3)	4.32
3	h	steps	-40	40	6.00E+1 (mm)	3.32E+01 (mm)	-33.52
4	K_{x1}	spring support	-75	75	1.00E+4 (KN/m)	7.74E+03 (KN/m)	-22.62
5	K_{x2}	spring support	-75	75	1.00E+4 (KN/m)	6.35E+03 (KN/m)	-36.52
6	K_{x3}	spring support	-75	75	1.00E+4 (KN/m)	1.02E+04 (KN/m)	1.95
7	K_{x4}	spring support	-75	75	1.00E+4 (KN/m)	7.52E+03 (KN/m)	-24.82
8	K_{y1}	spring support	-75	75	5.00E+2 (KN/m)	4.97E+02 (KN/m)	-0.59
9	K_{y2}	spring support	-75	75	5.00E+2 (KN/m)	5.00E+02 (KN/m)	0.06
10	K_{z1}	spring support	-75	75	1.00E+4 (KN/m)	9.64E+03 (KN/m)	-3.62
11	K_{z2}	spring support	-75	75	1.00E+4 (KN/m)	1.22E+04 (KN/m)	21.51
12	K_{z3}	spring support	-75	75	1.00E+4 (KN/m)	9.91E+03 (KN/m)	-0.94
13	K_{z4}	spring support	-75	75	1.00E+4 (KN/m)	1.17E+04 (KN/m)	16.66
14	K_{z5}	spring support	-75	75	1.00E+4 (KN/m)	1.41E+04 (KN/m)	41.05

Table 4. Values of starting and updated parameters

Mode number	Modal testing f_I [Hz]	Updated FE model f_{III} [Hz]	Difference $(f_{III} - f_I)/f_I$ [%]	Mode shape correlation MAC [%]
1B	6.37	6.36	-0.10	98.9
2B	18.63	18.64	0.05	95.1
1T	29.03	29.05	0.06	82.1

Table 5. Correlation between experimental and updated FE model



1B 2B 1T

Figure 14. Overlaying of mode shapes obtained experimentally (dotted line) and numerically in the final FE model (solid line)

8. Discussion

Although a very detailed initial FE model of the staircase structure was developed based on design data available and best engineering judgment, the discrepancies in the natural frequencies of the two first bending modes and first torsional mode were quite large between the experimental and first numerical results.

This initial FE model could not be updated in a physically meaningful way by using a sensitivity-based procedure implemented in the FEMtools updating software. Therefore, the manual tuning was required to minimise the difference between the initial FE model and its experimental counterpart before implementing the automatic updating.
For the staircase under study, the two screws in the side support were not able to prevent rotation of the structure about x-axis. Allowing for some rotation of this connection was crucial for success of tuning. With regard to the vertical stiffness of the supports at the top and the bottom of the stairs, it is interesting that the results of the updating procedure suggest that the top floor is stiffer than the bridge at the bottom. This is expected given the geometry of the structure.
Finally, to study the influence of the handrail on dynamic properties of the structure, the first two bending modes and first torsional mode of vibration of the staircase have been calculated after removing the handrail from the updated FE model presented in Table 5. The natural frequencies and MAC values obtained for the staircase FE model without the handrail are presented in Table 6, together with the natural frequencies obtained in Table 5. It can be seen that the frequency difference increased and MAC value decreased. Particularly poor mode shape correlation was obtained for the torsional mode.

Mode number	Modal testing f_I [Hz]	Updated FE model f_{III} [Hz]	FE model without handrail f_{IV} [%]	Difference $(f_{IV} - f_I)/f_I$ [%]	MAC [%]
1B	6.37	6.36	6.52	2.31	98.8
2B	18.63	18.64	19.49	4.65	90.1
1T	29.03	29.05	28.82	-0.72	53.9

Table 6. Influence of absence of the handrail

9. Conclusions

Using Finite Element modelling technique for numerical prediction of modal properties (natural frequencies and mode shapes) of the staircase structures, there is no guarantee that the initial model can estimate the modal properties of the staircase reasonably well, even when it is very detailed. Two first bending modes and first torsional mode of vibration of the staircase structure located in the Zeeman building on the University of Warwick Campus were identified via ambient vibration testing. A comparison with their estimates from the initial FE model revealed errors in the natural frequencies.
A manual tuning of the initial FE model was required to reduce the difference between the experimental and numerical results. Adding flexibility to the side support and both ends of the staircase improved considerably the correlation between the numerical and the experimental models. Only then the numerical model was possible to automatically update via the FEMtools software.
The updating procedure improved the frequency correlation and increased MAC values by changing the values of 14 uncertain parameters. The parameter changes suggested that the composite steps in the staircase were less stiff than assumed.

10. Acknowledgements

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