UPDATING FINITE ELEMENT MODELS USING FRF CORRELATION FUNCTIONS

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ABSTRACT

Updating finite element models directly from experimental Frequency Response Functions (FRF), instead of using modal parameters, has been the subject of extensive research in recent years. This paper presents a new approach, which is based on maximizing the correlation between analytical and experimental FRFs in terms of functions that describe the FRF shape and amplitude correlation over a frequency band.

The proposed method offers several advantages over alternative approaches related to the choice of reference frequencies, computational overhead, and convergence characteristics. In addition, it was found that updating results are fairly insensitive to noise on the measured FRFs. Implementation and application of the method is using an iterative scheme and Bayesian parameter estimation similar to what has already been successfully applied in modal-based updating methods. Somel examples are included to demonstrate the benefits of this new method.

1. INTRODUCTION

Frequency response function methods directly employ the measured quantities for updating and therefore circumvent tedious modal parameter extraction in a previous step. Theoretically, if noise-free measurements are available, the measured frequency response function represents an equation of motion at each frequency point (spectral line) that governs the dynamic behavior of the system precisely. It fully characterizes the system and contains in particular all the damping properties, which on the other hand must be modeled in the modal model, and hence approximated. Inherent truncation errors of modal models are not present in the measured FRFs as the effect of all modes is truly contained in the acquired data.

A variety of different approaches to FRF-based updating can be found in literature. An overview is given in Friswell and Mottershead [1]. In general, these methods rely on a selection of frequency points at which a residual is evaluated and then minimized. It was observed that due to properties of the FRFs, the use of a Taylor series expansion with first-order sensitivities may lead to divergent behavior depending on the choice of frequency points. Furthermore, the choice of frequency points also influence the updating results in the presence of noisy test data. The success rate of these methods is therefore largely dependent on the optimal choice of frequency points, which reduces their practical applicability.

It is not correct to assume that the error on the first-order differential FRF sensitivities is constant over the frequency range of interest. On the contrary, differences between differential and finite difference sensitivities are very variable over the frequency range and can become very large [2]. This result partially explains why one selection of frequency points will lead to convergence while another selection may cause divergence. Moreover, at different iteration loops, the shape of FRFs change and thus a good selection of frequencies points may be a bad selection in the next iteration. This explains unstable convergence when the selection of frequency points is kept identical for each iteration.

In a newly proposed FRF-based updating method [2,3], the problem of selecting a number of frequency points is avoided. In this method, all frequency points are considered and weighted according to the level of agreement with the measurements. The error residual that needs to be minimized is derived from the correlation between all available analytical and experimental FRFs, expressed in terms of shape correlation and amplitude correlation. The correlation is evaluated at all frequency points instead of at a number of selected reference frequencies. This has an averaging effect so that ‘bad’ choices are compensated by ‘good’ ones. Therefore, convergence stability, even in the presence of noise test data, benefits from this approach while at the same time making practical application of FRF-based updating much easier.

2. FRF CORRELATION FUNCTIONS

A measure of correlation between FRFs, equivalent to the Modal Assurance Criterion (MAC), is adapted for the frequency domain. At each frequency point ω_k, the level of correlation between all corresponding experimental FRFs (α_X) and predicted FRFs (α_A) is evaluated as

$$\mathrm{CSAC}(\omega_k) = \frac{\left|\alpha_X^H(\omega_k)\alpha_A(\omega_k)\right|^2}{\left(\alpha_X^H(\omega_k)\,\alpha_X(\omega_k)\right)\left(\alpha_A^H(\omega_k)\,\alpha_A(\omega_k)\right)} \quad (1)$$

with k = 1,2, ... N_f the number of frequency points. CSAC stands for Cross Signature Assurance Criterion. This criterion function is a measure of *shape correlation* with values ranging between 0 and 1. Because the shape of an FRF is mainly determined by the position and number of resonance peaks, this function is most sensitive to changes of mass and stiffness modeling.

To completely characterize the correlation between FRFs, it is necessary to introduce a second correlation function that evaluates discrepancies between *amplitudes*. This function is referred to as Cross Signature Scale Factor or CSF and is defined as

$$\mathrm{CSF}(\omega_k) = \frac{2\left|\alpha_X^H(\omega_k)\,\alpha_A(\omega_k)\right|}{\left(\alpha_X^H(\omega_k)\,\alpha_X(\omega_k)\right)+\left(\alpha_A^H(\omega_k)\,\alpha_A(\omega_k)\right)} \quad (2)$$

Like CSAC, the values of CSF can range between 0 and 1. Because CSF evaluates amplitude, this function is more sensitive to changes of damping.

Together, these two FRF correlation functions are referred to as the Cross Signature Correlation (CSC) functions.

3. FRF CORRELATION SENSITIVITY ANALYSIS

The sensitivity coefficients for FRF correlation functions CSAC and CSF are obtained by taking the differentials of equations (1) and (2) with respect to a parameter p. If this is done for N_p parameters, then a sensitivity matrix [S] is obtained with dimension $2N_f$ x N_p. There are twice as many equations as there are frequency points in the frequency range of interest.

In the general case of a damped structure, the sensitivities are functions of the analytical and experimental FRFs but also of the differentials of the real (Re) and imaginary (Im) parts of the analytical FRFs [2, 3]:

$$\frac{\partial \mathrm{CSAC}(\omega_k)}{\partial p} = f'(\alpha_X, \alpha_A, \frac{\partial\,\mathrm{Re}(\alpha_A)}{\partial p}, \frac{\partial\,\mathrm{Im}(\alpha_A)}{\partial p}) \quad (3)$$

$$\frac{\partial \mathrm{CSF}(\omega_k)}{\partial p} = f''(\alpha_X, \alpha_A, \frac{\partial\,\mathrm{Re}(\alpha_A)}{\partial p}, \frac{\partial\,\mathrm{Im}(\alpha_A)}{\partial p}) \quad (4)$$

if

$$\alpha_{ij} = \mathrm{Re}(\alpha_{ij}) + \mathrm{Im}(\alpha_{ij})i \quad (5)$$

The sensitivities are not purely analytical but include experimental data. Therefore, one could refer to them as mixed analytical-experimental sensitivities. Moreover, unlike α_{ij} or $\frac{\partial\alpha_{ij}}{\partial p}$, they are real values which makes them more easy to interpret.

The differentials of α_A are obtained as

$$\frac{\partial\alpha_A}{\partial p} = \frac{\partial(\alpha_A\,[Z]\,\alpha_A)}{\partial p} = -\alpha_A\,\frac{\partial[Z]}{\partial p}\,\alpha_A \quad (6)$$

Note that the receptance matrix α_A is the inverse of the dynamic stiffness matrix:

$$[\alpha_A] = \frac{\{U\}}{\{F\}} = [Z]^{-1} \quad (7)$$

with

$$[Z] = [K] + i\omega[D] - \omega^2[M] \quad (8)$$

Solving equation (6) at each frequency point can become unfeasible for larger systems. However, usually a modal solution is available, and it is much more efficient to apply a modal transformation to approximate α_A as

$$[\alpha_A(\omega_k)] \approx [\Phi]\left[\omega_r^2 - \omega_k^2 + 2i\omega_r\omega_k\zeta_r\right]^{-1}[\Phi]^t \quad (9)$$

where $[\Phi]$ represents the modal matrix (mode shapes) normalized with respect to the system mass matrix, ω_r are the resonance frequencies and ζ_r is the viscous damping ratio commonly used to specify the amount of damping as a percentage of the critical damping.

4. FE MODEL UPDATING USING FRF CORRELATION FUNCTIONS

The validity of a finite element can be evaluated as a function of how well experimentally obtained frequency response functions correlate with the ones predicted using the finite element model. Updating the FE model in this context means changing the structural behavior, in terms of mass, stiffness and damping characteristics, in such a way that the CSAC and CSF functions converge to 1 in the frequency range of interest. Mathematically, the following error function needs to be minimized [4]:

$$E = \Delta\varepsilon^t C_R \Delta\varepsilon + \Delta p^t C_p \Delta p \qquad (10)$$

with

$$\Delta\varepsilon = \begin{Bmatrix} 1 - CSAC(\omega_k) \\ 1 - CSF(\omega_k) \end{Bmatrix} \qquad (11)$$

and p the selected updating parameters. This can be a material property, plate thickness, lumped mass, joint stiffness or any other physical element property.

C_R and C_p are respectively diagonal weighting matrices for the selected FRF correlation function values and for the updating parameter values. Each weight may be considered as the level of confidence one has in the value.

Solving (10) for the unknown design changes, in case $2N_f > N_p$, yields:

$$\Delta p = \left[[C_P] + [S]^t[C_R][S]\right]^{-1}[S]^t[C_R]\Delta\varepsilon \qquad (12)$$

The sensitivity matrix [S] is

$$[S] = \begin{Bmatrix} \dfrac{\partial CSAC(\omega_k)}{\partial p} \\ \dfrac{\partial CSF(\omega_k)}{\partial p} \end{Bmatrix} \qquad (13)$$

5. DISCUSSION

Using FRF data directly offers several advantages over using modal parameters. FRFs are not subject to curve-fitting errors and provide information on damping characteristics over the entire measured frequency spectrum. Several FRF-based methods have been presented in literature over the last decade but most of them fail in practice due to excessive computations, or to numerical instability. Both types of problems can be related to choice of frequency points. The new method presented herein has been designed to overcome these drawbacks and turn FRF-based updating into a practical, efficient and easy-to-use complement to modal-based updating methods.

The selection of frequency points used in the updating equations, often a crucial and difficult issue in alternative methods, is avoided in the present method. Simply the frequency range must be selected and all frequency points in this range are used in the updating procedure. However, this does not exclude that filters be added to exclude frequency points. This may be to

- Enhance stability - Too many spectral lines on or near resonance frequencies can lead to numerical instability.
- Focus on damping – Only frequency points around resonance frequencies are effective to update damping.
- Reduce computation time – If a small frequency step was used, including all frequency points yields too many redundant data and requires unnecessary calculations.

The advantage of using many frequency points is that 'bad' choices are compensated by 'good' choices. Bad choices are frequency points for which the differential FRF amplitude sensitivity contains too much error, or the test data is too noisy.

Although the updating results benefit from adding more FRFs, this does not increase the number of frequency points, unlike traditional FRF-based updating methods. Indeed, the number of updating equations $2N_f$ is only determined by the number of frequency points in a selected frequency range, and not by the number of FRFs. Therefore, adding more FRFs to the database does not change the calculation time required for updating, but has only minor influence on the calculation times required for FRF correlation analysis, and sensitivity analysis.

When a modal transformation is used to compute FRF correlation sensitivities (equation 9), a modal eigensolution is required with each iteration loop. It is important to include a sufficient number of modes in the modal transformation (highest resonance frequency > 2 X highest frequency in the range). It is also useful to limit the parameter modification per iteration loop because first-order sensitivities are used.

The analytical FRFs are usually synthesized from the normal modes of the structure. These FRFs too suffer from truncation and the measured FRFs do not. The FRF correlation functions are more heavily affected by the larger amplitudes, which in turn are mainly caused by modes.

They are less sensitive to anti-resonances, which are significantly smaller but heavily affected by truncation. Therefore, the costly direct computation of FRFs instead of using synthesis is not as critical here as they are in the classical FRF updating approaches using spectral lines.

Other equally important considerations that are not further discussed here are the :

1. number of computed normal modes vs. the FE mesh density

2. spatial distribution and number of pickup sensors

Although the proposed updating method allows for simultaneous updating of mass, stiffness and damping, this approach is seldom successful. Using a 2-step procedure in which first mass and stiffness are updated (with highest impact on CSAC) and then subsequently updating of damping (with highest impact on CSF) has proven to work better in practice.

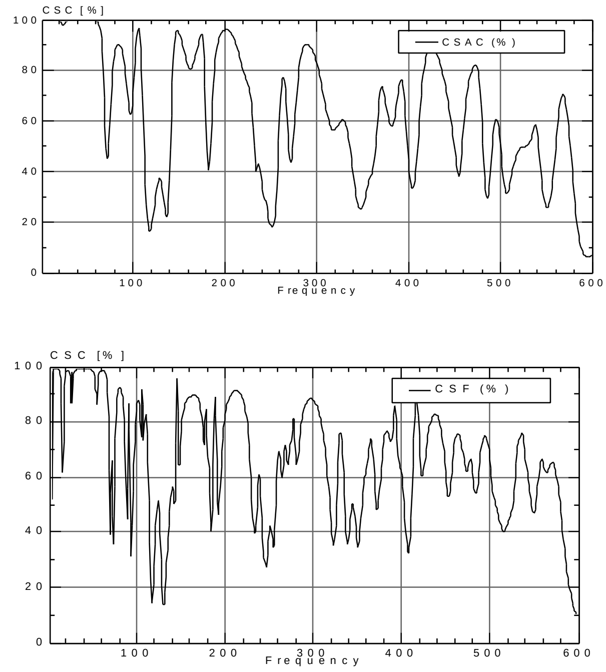

Figure 1. Initial CSAC (top) and CSF (bottom) function for 0-600 Hz.

6. APPLICATION EXAMPLES

To demonstrate the application of the proposed method, important stiffness modeling errors in a plate model are simulated. Using the normal modes of the altered FE model, several FRFs are synthesized that serve as reference data. Updating was done by selecting CSAC and CSF values in the 50-250 Hz band with a frequency step of 1 Hz. The FEMtools updating software was used for all analysis [5].

Figure 1 shows that the initial FRF-correlation conditions show CSAC and CSF values that average around 60%.

Figure 2 shows the updating results after 10 iterations. Convergence is indicated as the average residual error between the actual values of the CSAC and CSF functions and their target values (= 1). Convergence is almost complete after 10 iterations.

Figure 3 shows the correlation results when updating is done (10 iterations) in the 50-250 Hz band.

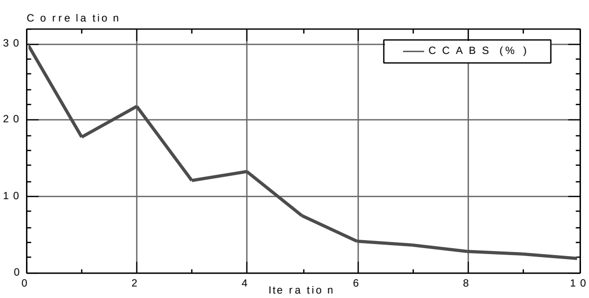

Figure 2. Average residual error of CSAC and CSF as function of iteration loop.

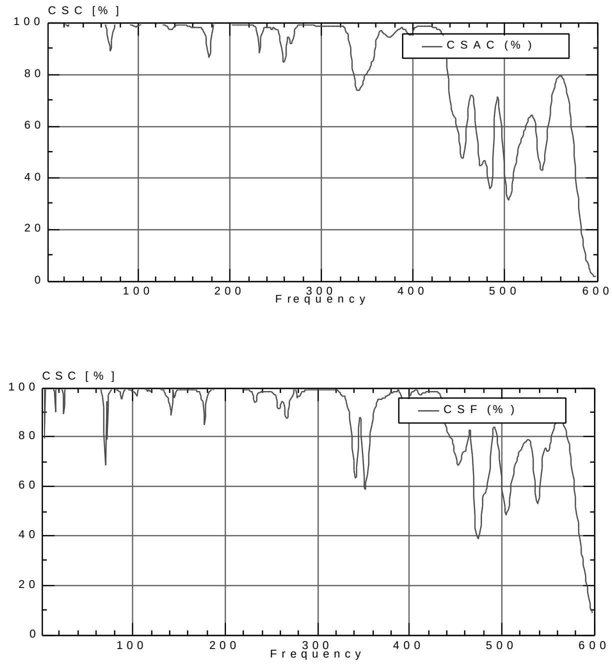

Figure 3. CSAC (top) and CSF (bottom) function after model updating in the 50-250 Hz band.

In the previous test case, updating was done with idealized noise-free test data. Now 15% multiplicative noise is added to the reference FRF data and the calculations repeated under the same conditions. The polluted response vector $\{\alpha\}$ is obtained by the following operation

$$\{\alpha\} = \{\alpha\}(1 + k\{\text{noise}\}) \tag{14}$$

with k the percentage multiplicative noise and {noise} a vector of random numbers between 0.0 and 1.0.

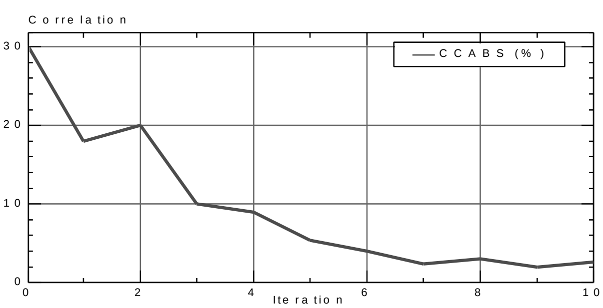

Figure 4. Average residual error of CSAC and CSF as function of iteration loop in case of noisy test data.

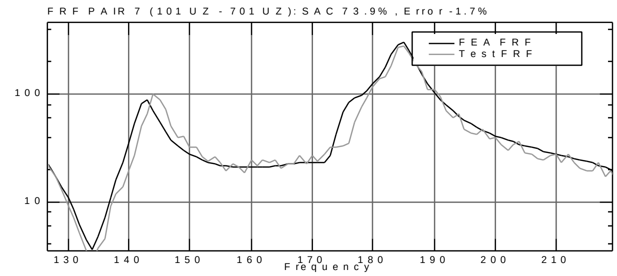

Figure 5. Detail of a test-FEA FRF pair after updating using noisy test data.

It can be concluded that the updating method behaves remarkably well even when noisy test data is used (figure 4). However, as could be expected, complete convergence can no longer be obtained. The synthesized analytical FRFs are to be considered as smoothed curve-fits of the noisy experimental FRFs (figure 5).

7. CONCLUSIONS

Two expressions are defined to measure the correlation over a given frequency range between a set of experimental FRFs and their analytical counterparts. These functions are used in a sensitivity-based updating scheme that tends to maximize their values by iteratively changing a set of updating parameters.

Verification analysis using simulated test data has shown that the method leads to stable convergence, and behaves well in the presence of noisy test data. In addition, the method is extremely easy to use and, if a modal transformation is used for sensitivity analysis, very cost-efficient even if many frequency points are used. The computation time is not much depending on the number of FRFs that are included in the analysis.

In cases where using local information is important, only a few FRFs are available, and the frequency band of interest is narrow, the traditional methods may be preferable. In all other cases, the newly proposed method shows clear advantages. To test the industrial applicability, the method has been applied successfully on large FE models with real test data. However, more application experience is still required. Future papers will report on this.

The limitations of the current approach are mainly related to the use of first-order differential sensitivities computed using modal transformation of the dynamic stiffness matrix. These sensitivities are only valid for small parameter perturbations, require that a sufficiently high number of eigenmodes is taken into account and that the eigenmodes do not vary much due to the parameter perturbation. Off course, these conditions are more difficult to satisfy when large parameter errors are required to improve FRF correlation. At the cost of doing more iterations, forcing small parameter changes per iteration loop is one approach to overcome this situation. However, like all gradient-based minimization schemes, this does not exclude the possibility that a local minimum is found rather than the global one. Alternatively, using a direct solution for sensitivity analysis instead of a modal transformation, using higher-order sensitivities, using logarithmic sensitivities or altogether using a different minimization scheme, are topics for future work that hold promise to further enhance the performance of the method.

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