PREDICTION OF THE VIBRATION IN BUILDINGS USING STATISTICAL ENERGY ANALYSIS

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Abstract

The purpose of this paper is to establish a method of predicting the vibration within buildings in the co-generator operating state by using Statistical Energy Analysis (SEA). An analytical model of a building is constructed, and power flow equations are formulated for the building. To solve these equations, SEA parameters are estimated experimentally and analytically. These parameters are the modal density, intrinsic loss factor, coupling loss factor, and input power. In particular, the intrinsic loss factor of the structural elements is estimated using the power spectrums of the driving-point, which represent the characteristics of the structural elements, and the real part of the mobility is calculated with the FEM. With these parameters, the vibration responses in the rooms are calculated. The results, confirmed that the method validly predicted the vibration level in the building by considering the sound location sub-systems.

Nomenclature

P_{i1}	: power input to sub-system 1
P_{l1}	: power intrinsic loss of sub-system 1
P_{12}	: power transmitted from sub-system 1 to sub-system 2
E_1	: average energy of sub-system 1
ω	: band center angular frequency
η_1	: intrinsic loss factor of sub-system 1
η_{12}	: coupling loss factor from sub-system 1 to sub-system 2
n_1	: modal density of sub-system 1
N_1	: mode count of sub-system 1
M	: mass of sub-system
$\langle v^2 \rangle$	: spatial square average of the vibration velocity
$\langle p^2 \rangle$	: spatial square average of the sound pressure
Z_o	: specific acoustic impedance of air
A	: area of sub-system
t	: thickness of sub-system
ρ	: mass density
ν	: Poisson's ratio
E	: Young's modulus
V	: volume of the sound location sub-system
c	: speed of sound
$\mathrm{Re}(Y)$	: real part of mobility
F^2	: power spectrum of vibration force
v^2	: power spectrum of response velocity
$\underline{S}$	: surface area of the sound location sub-system
α	: acoustic absorption coefficient in mean form
c_{gi}	: group velocity of the bending waves
S_i	: surface area of sub-system
τ	: energy transmission efficiency
σ	: acoustical radiation efficiency
L_c	: coupled length between structural sub-system
S_c	: coupled surface area between structural and acoustical sub-system

1. Introduction

In recent years, at buildings such as hospitals, a co-generation system is used for heat efficiency improvement from the viewpoint of the valid energy utilization. Although the engine was located previously, in the basement or rooftop, recently, the engine has been placed in the middle layer as well. Therefore, the problem of noise and vibration is important in the neighborhood of the engine room, and a method of predicting the noise and vibration in the room is thus required. However, when dealing with the high frequency band, it is difficult to predict sound and vibration using the traditional technique.

In this research, we adopt a building as the object of the research, and estimate the vibration response of the building using the Statistical Energy Analysis (SEA) method. This building is composed of special rooms, including a semi-anechoic room. More over, the 1st floors are thick compared with the other walls. In previous publications, a building that consists of such a complicated construct has not been considered.

The SEA method was developed by Lyon et al. as a noise and vibration estimation technique for complex solid structures that include integration with sound locations [1].

With the SEA method, we consider structural components as a set of equivalent vibrating elements, and evaluate the vibration condition of the components as a macroscopic statistical average quantity versus the frequency band and space (by using the energy) by assuming that certain vibration modes within some frequency bands are uniformly distributed and are excited to the same degree. Merits of the SEA method are as follows [2]: (1) because the energy is used as the conditional quantity of the structure, the structural components and sound location components can be expressed together, and this simplifies the equation for the energy transmission between the components; (2) because the energy is assumed as the fundamental factor, the flow of vibration energy can be easily understood, and parts requiring measures to be taken are understood at a glance; and (3) because statistical considerations are made, estimation is possible even when the number of certain vibration modes within the object frequency band is several hundreds or several thousands. Therefore, we believe that the method is extremely effective for the purpose of our research.

2. Power Flow Equations

With the SEA method, we can formalize the relationships of the power flows between sub-systems, and by solving these relationships, we can calculate the energy held by each of the sub-systems. Below, an explanation is given on such basic power flow equations [3].

2.1 Power Flow Equations of a Two Sub-System Structure

The power flow relationships of a structure consisting of a two sub-system structure are shown in Figure 1. The equations for the power flows between sub-systems 1 and 2 under typical conditions are expressed as follows.

$$\text{Sub-system 1: } P_{i1} = P_{l1} + P_{l2} \tag{1}$$

$$\text{Sub-system 2: } P_{i2} = P_{l2} + P_{21} \tag{2}$$

where power intrinsic loss P_{l1} becomes

$$P_{l1} = \omega\eta_1 E_1 \tag{3}$$

In addition, power transmitted P_{12} is expressed with the following equations.

$$P_{12} = -P_{21} = P'_{12} - P'_{21} \tag{4}$$

$$P'_{12} = \omega\eta_{12}E_1, P'_{21} = \omega\eta_{21}E_2 \tag{5}$$

where η_{12} and η_{21} in SEA satisfy the reciprocity relationship $\eta_{12}n_1 = \eta_{21}n_2$. Therefore, transmitted power P_{12} becomes

$$P_{12} = \omega\eta_{12}\left(n_2E_1 - n_1E_2\right) = \omega\eta_{12}n_1\left(\frac{E_1}{n_1} - \frac{E_2}{n_2}\right) \tag{6}$$

Consequently, power flow equations (1) and (2) can be expressed as follows.

$$P_{i1} = \omega\eta_1E_1 + \omega\eta_{12}n_1\left(\frac{E_1}{n_1} - \frac{E_2}{n_2}\right) \tag{7}$$

$$P_{i2} = \omega\eta_2E_2 + \omega\eta_{21}n_2\left(\frac{E_2}{n_2} - \frac{E_1}{n_1}\right) \tag{8}$$

From this, if the SEA parameters, i.e., the modal density, intrinsic loss factor, coupling loss factor, and input power, are given, the energy condition of each sub-system can be easily calculated.

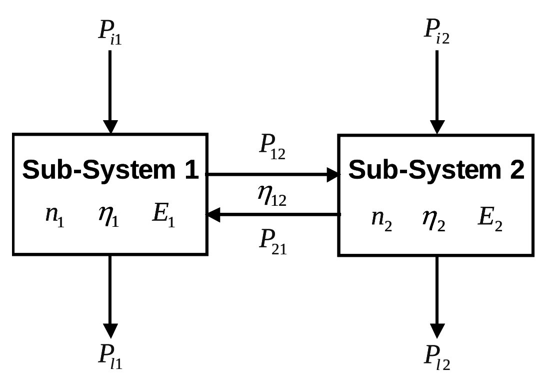

Figure 1: Power flow relationships between two sub-systems

2.2 Power Flow Equations of a Multiple Sub-System Structures

By expanding our way of thinking in the previous section, it is possible for us to formalize, in the same way, the power flow relationships of a structure made up of multiple sub-systems. The power flow equation for a structure made up of N sub-systems is expressed with the following equation in matrix form.

$$\omega \begin{bmatrix} \left(\eta_1 + \sum_{i \neq 1}^{N} \eta_{1i}\right) n_1 & -\eta_{12} n_1 & \cdots & -\eta_{1N} n_1 \\ -\eta_{21} n_2 & \left(\eta_2 + \sum_{i \neq 2}^{N} \eta_{2i}\right) n_2 & \cdots & -\eta_{2N} n_2 \\ \vdots & \vdots & \ddots & \vdots \\ -\eta_{N1} n_n & \cdots & \cdots & \left(\eta_N + \sum_{i \neq N}^{N-1} \eta_{Ni}\right) n_N \end{bmatrix} \times \begin{bmatrix} E_1 / n_1 \\ E_2 / n_2 \\ \vdots \\ E_N / n_N \end{bmatrix} = \begin{bmatrix} P_{i1} \\ P_{i2} \\ \vdots \\ P_{iN} \end{bmatrix} \tag{9}$$

From Eq. (9), if the SEA parameters are given in the same way as for the above two sub-system structure, the energy condition of each sub-system can be obtained.

The energy of a sub-system, assuming an average linear sub-system, is expressed with the following equations by using the vibration acceleration and sound pressure.

$$E = M \left\langle v^2 \right\rangle \tag{10}$$

$$E = \frac{M \left\langle p^2 \right\rangle}{Z_o^2} \tag{11}$$

where M is the mass of the sub-system, $\left\langle v^2 \right\rangle$ is the spatial square average of the vibration velocity, $\left\langle p^2 \right\rangle$ is the spatial square average of the sound pressure, and Z_o is the specific acoustic impedance of air.

Accordingly, if the energy condition of each component is determined from Eq. (9), it is possible to calculate the vibration condition and the sound pressure by using Eqs. (10) and (11).

2.3 Modeling of the Building

In this research, the building in which the vibration level in each room was measured is a reinforced-concrete laboratory. The configuration of the building is shown in Figure 2. This building is complicated because it is composed of seven rooms and contains one semi-anechoic room. The floor of the semi-anechoic room is composed of a thick part that is called bench; a generator that is installed on it is operating under the idle state (2700 rpm/min). If an estimation method for the vibration level under the idle state were to be established, then it would also be applicable under other conditions.
In the analysis, it is assumed that the material of the structure was dense concrete with a Young's modulus of 2.7E+10 N/m^2, a mass density of 2300 kg/m^3, and a Poisson's ratio of 0.17. Here, because we have assumed analysis with the SEA method, the SEA model of the slab that is composed of benches (composed of six sub-systems) is constructed to identify the input power. After solving the SEA model, which is composed of six sub-systems, the entire model of the building is built to estimate the vibration in each sub-system. Therefore, this system of the building is composed of 44 sub-systems. Then, because a mass of window and door that exist on a concrete wall is very small in comparison, we removed them and their calculated equivalence thickness. Here, in all the sub-systems, sub-systems 1 to 37 are structural components, and sub-systems 38 to 44 are acoustical components, respectively.

Figure 2: The configuration of the building

Figure 3 can be considered to show the example of the power flow relationships between two rooms within the building. The sub-systems connected by lines in Figure 3 are joined sub-systems. Here, the different line types show the I-type, L-type, and T-type combination, respectively.

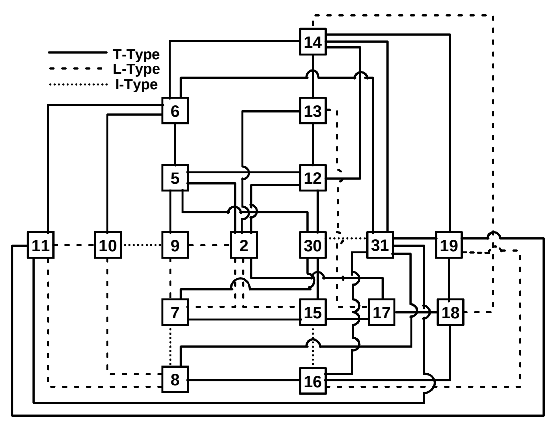

Figure 3: Example of the power flow relationships

within the building

3. Estimation of SEA Parameters

In order to solve the power flow equations, it is necessary to determine the SEA parameters, i.e., the modal density, intrinsic loss factor, coupling loss factor, and input power. Below, an explanation is given of a method of calculating SEA parameters for the building.

3.1 Modal Density

Mode count N, included in the frequency band (for estimation), is a ratio denoting how easily energy, in the exchanging of energy between sub-systems, can be obtained [2]. In order to determine the value of N in the prescribed frequency band, it is first necessary to determine modal density $n(f)$, i.e., the gradient of N in the frequency band.

The modal density of structural sub-systems is the following equation [4].

$$n = \frac{A}{4\pi t}\sqrt{\frac{12\rho\left(1-\nu^2\right)}{E}} \tag{12}$$

where A is the area, t is the thickness of the structural sub-system, ρ is the mass density, ν is the Poisson's ratio, and E is the Young's modulus.

Figure 4 shows the results of an example in which the modal density of each sub-system was obtained by using Eq. (12).

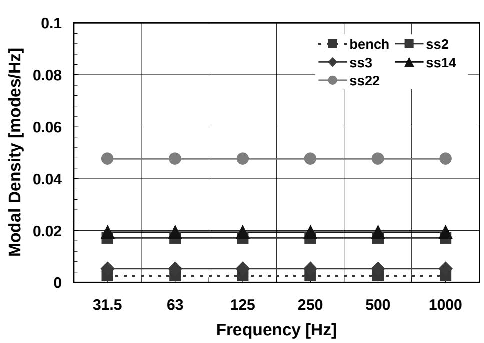

Figure 4: Modal density of structural sub-systems

In addition, the modal density of the sound location sub-system is determined by the following equation, since it is difficult to measure.

$$n(f) = \frac{4\pi V}{c^3} f^2 \tag{13}$$

where c is the speed of sound within the sound location sub-system, and V expresses the volume of the sound location sub-system.

3.2 Intrinsic Loss Factor (ILF)

The intrinsic loss factor η_1 of a sub-system gives the loss percentage when the input power to the sub-system from the outside is converted to the motion energy of the sub-system. Clarkson et al. proposed the following equation as a test equation to determine the ILF of structural sub-systems in vibration excitation experiments [5].

$$\eta = \frac{\int_{f_1}^{f_2} \mathrm{Re}(Y) F^2 df}{\omega_o M \left\langle \int_{f_1}^{f_2} v^2 df \right\rangle} \tag{14}$$

where Y is the driving-point mobility in the range of f_1 to f_2, F^2 is the power spectrum of the added vibration force, and v^2 is the power spectrum of the response speed. In addition, $\langle\ \rangle$ indicates the space average.

In this research, it is difficult to give effect to the excitation test in the 'free' support condition to each structure element at the actual building. Therefore, the ILF of the structural sub-system is estimated by using the power spectrum of a driving-point (this point divides a diagonal line into 7:3), which represents the characteristic of the concrete slab [6] and the real part of the mobility calculated using the analytical software FEM tools.

The real part of the mobility of the structural sub-system is expressed as below.

$$\mathrm{Re}(Y) = \frac{2\Omega_r \phi_{rr}^2 \varsigma_r \beta_r^2 / k_r}{\left(1-\beta_r^2\right)^2 + \left(2\varsigma_r \beta_r\right)^2} \tag{15}$$

where Ω_r is the undamped natural angular frequency of the r^{th} mode, ϕ_{rr} is the r^{th} element of the r^{th} mode shape, and k_r is the modal stiffness of the r^{th} mode. Here, if $\beta_r = \omega/\Omega_r$, then the ζ_r is 0.03. This is the general value for the damping ratio of the concrete structure.

Comparing the FEM and the experimental modal analysis in grounded support condition, the accuracy of the FEM was validated. Then, the modal analysis was performed in 'free' support condition using this FEM to calculate the real part of the mobility in the structural sub-system.

Figure 5 shows the result of the ILF of the structural sub-system obtained through an octave analysis by using

Eq. (14).

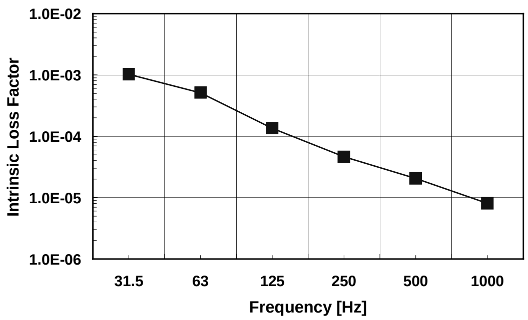

Figure 5: Intrinsic loss factor of structural sub-system

The ILF of the sound location sub-systems is determined by the following equation, since it is difficult to measure the ILF.

$$\eta = \frac{cS\overline{\alpha}}{4V\omega} \tag{16}$$

where $\overline{\alpha}$ is the acoustic absorption coefficient in average form, and S denotes the surface area of the sound location. The mean absorption coefficient was estimated from Sabine's equation using the reverberant time, by the square-integration method, which used the impulse excitation sound. The reverberation time of a room was estimated at 0.41 s.

Figure 6 shows the result of the ILF of the sound location sub-system obtained through an octave analysis using Eq. (16).

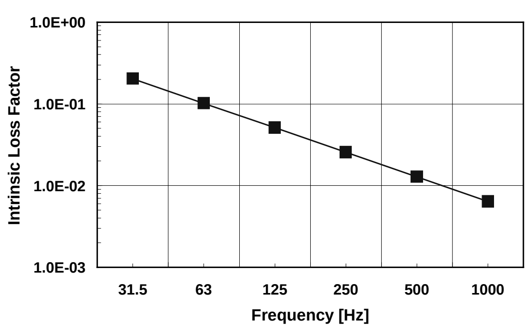

Figure 6: Intrinsic loss factor of acoustical sub-system

3.3 Coupling Loss Factor (CLF)

The coupling loss factor gives the loss percentage when power transmits between sub-systems. Because it is difficult to measure the CLF between sub-systems in the building, we use the following equations that show the CLF between structural sub-systems, and the CLF between a structural sub-system and sound location sub-system, respectively [1].

$$\eta_{ij} = \frac{c_{gi} L_c \tau}{\pi \omega S_i} \tag{17}$$

$$\eta_{ij} = \frac{Z_o S_c \sigma}{\omega M_i} \tag{18}$$

where c_{gi} is the group velocity of the bending waves, S_i is the surface area, M_i is the mass, τ is the energy transmission efficiency, σ is the acoustical radiation efficiency, L_c is the coupled length, and S_c is the coupled surface area. The transmission efficiency differs depending on the coupled type, e.g., I-type, L-type, or T-type [7].

Figures 7 and 8 show the results of the CLF structural sub-system, and the CLF between a structural sub-system and sound location sub-system was obtained through an octave analysis by using Eqs. (17) and (18), respectively.

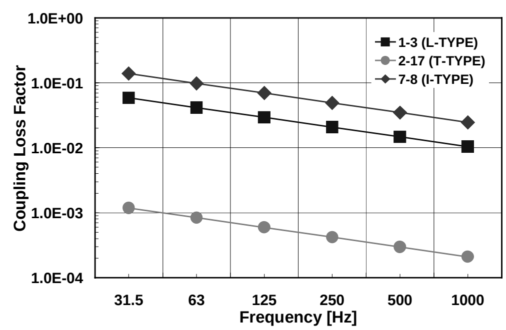

Figure 7: Coupling loss factor of structural sub-system

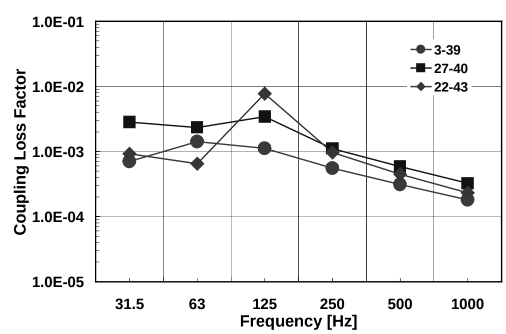

Figure 8: Coupling loss factor between a structural sub-system and sound location sub-system

3.4 Input Power

Input power P_{iN} expresses the power conveyed from the outside to each of the sub-systems.

In this research, because acoustical input power spreads insignificantly as structure borne sound compared with the vibration input power in the building, we do not consider it as the input power. Therefore, the vibration on the bench, which is directly propagated by the generator, is used as the ''vibration input power''.

The vibration input power is given by the following equation.

$$P_{iN} = \omega M_i \left\langle v_i^2 \right\rangle \qquad (19)$$

where ω is the band center angular frequency, M_i is the mass, and $\left\langle v_i^2 \right\rangle$ is the spatial square average of the vibration velocity.
In regard to the vibration velocity, we measured 15 points on the bench and calculated the spatial average.

4. Estimated By the SEA Method

By using the SEA parameters determined in previous section, we estimate the acceleration response of each sub-system with the SEA method.

4.1 Acceleration Response Estimation

We formulated power flow equations by using the power flow relationships for the building in Figure 3, and determined the acceleration response of each sub-system by introducing the SEA parameters. The obtained results are shown in Figure 9. An octave analysis was performed assuming the frequency range for the estimation to be from 31.5 Hz to 1000 Hz (where sound and vibration become problematic). In Figure 9, results are also shown in which acceleration responses were measured by attaching acceleration pick-ups to each sub-system while the generator was idling. In order to obtain the space averages of the measured values, the acceleration pick-ups were attached at from four to eight different locations for each sub-system, and the average was obtained.

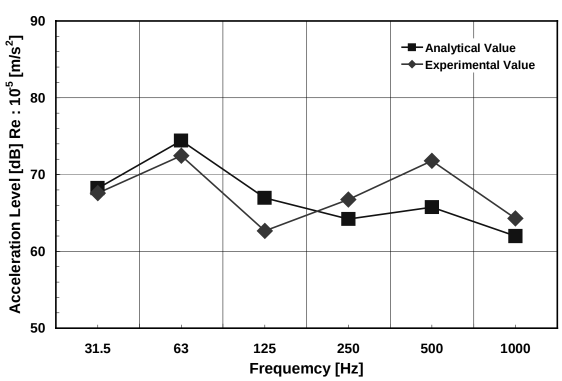

Figure 9a: Estimation results of the acceleration of structural component ss-4

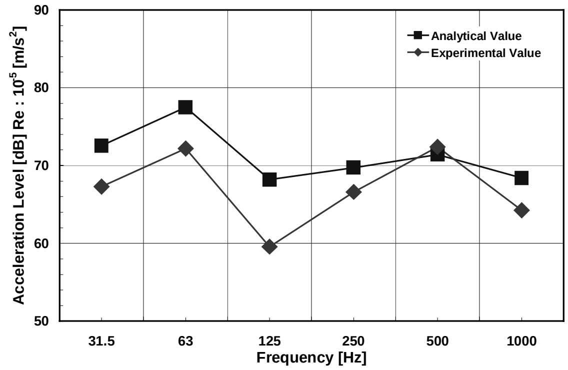

Figure 9b: Estimation results of the acceleration of structural component ss-9

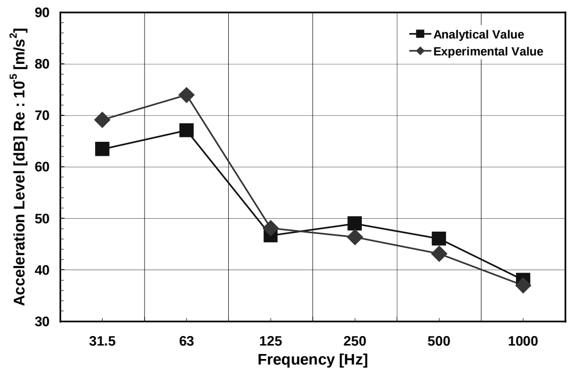

Figure 9c: Estimation results of the acceleration of structural component ss-25

5. Discussion

Each value in Figure 4 is the modal density of each sub-system. The wall and 2nd floor (ss-14, ss-22) are larger than the 1st floor or bench. Although the bench has the smallest modal density of all the sub-systems structuring the SEA model that is composed of benches, the modal density of ss-2 is about 6 times that of each bench.

As is apparent in Figures 5 and 6, the ILF of the structural sub-system and acoustical sub-system decreases as the frequency increases. Also, when looking at Figures 7 and 8, the CLF structural sub-systems, and the CLF between a structural sub-system and sound location sub-system decreases as the frequency increases, respectively. However, the CLF between a structural sub-system and sound location sub-system increases to a large number in the 63, 125 Hz band under the influence of the coincidence critical frequency of the structure sub-systems.

Sub-systems 4, 9, and 25 are in the 1st floors, the walls, and the 2nd floors, respectively. Table 1 indicates that the frequency band average differences dB of each sub-system in the 1st floors, the walls, and the 2nd floors, average 3, 2, and 3 sub-systems, respectively. Because sound absorption material was applied, we could not measure the vibration response in every wall. Because the differences data in the walls also contains the data in the semi-anechoic room wall, this number is large. Also, for the 1st floor data, by propagating the semi-anechoic wall as the dominant power path, the differences number increases. For the 2nd floors, good agreement is found between the analytical and experimental data.

Therefore, the SEA model for such a complicated building must be adjusted to account for floors that are thicker than the other sub-systems and the semi-anechoic room wall. In addition, because complicated buildings have many cavity walls, in many cases there will be fibrous quilt in the cavity that will add damping [8]. These influences must be considered further.

	31.5 [Hz]	63 [Hz]	125 [Hz]	250 [Hz]	500 [Hz]	1000 [Hz]
1st floors [dB]	4.28	4.25	12.1	8.10	8.06	11.9
walls [dB]	11.2	11.2	9.23	4.76	4.85	10.7
2nd floors [dB]	3.89	4.80	2.63	3.26	4.37	1.45

Table 1: The frequency band average differences of each sub-system

6. Conclusion

The results presented show that the technique of statistical energy analysis can be used to determine the vibration level of walls and floors in the complicated building.

1. The vibration input power in entire building systems (44 sub-systems) can be identified to solve the power flow, which is composed of a bench (6 sub-systems).
2. The intrinsic loss factor of the structural sub-system can be estimated by using the power spectrum of a driving-point, which represents the characteristic of the concrete slab and the real part of the mobility calculated using with the FEM.
3. By determining the SEA parameters (i.e., modal density, intrinsic loss factor, coupling loss factor, and input power) of each sub-system of the building, acceleration response estimation can be carried out by using SEA.

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