

# Validation of the Resonalyser method: an inverse method for material identification

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## Abstract

The Resonalyser method uses resonance frequencies measured on rectangular plate specimens to identify orthotropic material properties. An inverse technique is used to update the material properties in a numerical model of the test plate. The obtained material properties of steel and aluminum test plates are validated with the results of standard impulse excitation tests and standard tensile tests. Impulse excitation tests (IET) were performed on beam specimens cut in different material directions of the plates. IET uses in-plane and torsional vibration modes to identify the Young's moduli, shear moduli and Poisson's ratios in the orthotropic material axes and off-axis directions. It was found that the obtained results were situated well within the error intervals of the tensile test results and that the results from IET were in good agreement with the Resonalyser results. The error bounds of the Resonalyser tests have the same small magnitude as impulse testing. Both methods based on vibration measurements are accurate and produce repeatable results.

## 1 Introduction

Many engineering materials behave in an anisotropic manner: their response to external solicitations depends on the loading direction. A simple but common form of anisotropy is orthotropy. The elastic behavior of materials having orthotropic symmetry axes (e.g. rolled metal sheets or long-fiber reinforced composites), in a state of plane stress, can be described by the following relation between strains and stresses:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (1)$$

In relation (1),  $\{\varepsilon_i\}$  represents the strain components,  $\{\sigma_j\}$  the stress components,  $E_i$  the Young's modulus in the  $i$ -direction,  $\nu_{ij}$  the Poisson's ratios, and  $G_{12}$  is the shear modulus in the (1,2)-plane. If linear material behavior is assumed, the elastic properties  $E_i$ ,  $\nu_{ij}$  and  $G_{12}$  are also called the 'engineering constants'. Due to the symmetry of the compliance matrix in (1), only four (instead of five) independent engineering constants occur: e.g.  $E_1$ ,  $E_2$ ,  $\nu_{12}$  and  $G_{12}$ .

Knowledge of the elastic properties of materials is important for their use in structural applications, as well as for the improvement of the processes used to transform them into components. Elastic properties also play a major role in the vibration behavior of constructions. This observation can be inverted, leading to the conclusion that the vibration behavior of samples of a particular material can be used to determine the material's elastic properties.

The Impulse Excitation Technique (IET) is based on this observation. IET uses analytical formulas to calculate the elastic moduli from the resonance frequencies of a test beam, and serves as an alternative to the more traditional static tests in which a controlled load (or deformation) is applied while monitoring the resulting deformation (or load). Both IET and static tests (e.g. tensile tests) are standard-procedures for assessing the stiffness of an elongated test sample along its long axis (IET: ENV-843-2 and ASTM 1876, tensile tests: E111).

More recently, an inverse method, called "Resonalyser procedure", was developed to determine all four engineering constants for orthotropic materials from the resonance frequencies of rectangular plate samples [1]. In this paper, standard tensile and IET tests are used to validate the accuracy and sensitivity of the Resonalyser procedure for the case of rolled metal sheets.

## 2 Theoretical background

### 2.1 The "Poisson" test plate

The Resonalyser procedure is a mixed numerical-experimental method that aims to identify the engineering constants of orthotropic materials using measured resonance frequencies of freely suspended rectangular plates. Using rectangular plates as test specimen allows the simultaneous identification of  $E_1$ ,  $E_2$ ,  $\nu_{12}$  and  $G_{12}$ . In addition, the obtained elastic material properties are homogenised over the plate surface and hence suitable as input values for finite element models of structures. Also the amount of machining induced edge damage is reduced when using plate shaped rather than elongated specimens.

The basic principle of the Resonalyser is to compare experimentally measured frequencies with the numerically computed frequencies of a finite element model of the test plate. Such an inverse procedure can only yield good results if the numerical model is controllable and if the elastic properties can be observed through the measured data [2-3]. This requires that in the selected series of frequencies at least one of the frequencies varies significantly for variations of each of the elastic properties. It can be shown that this requirement is fulfilled if the length to width ratio of the test plate approximately complies with  $\frac{Length}{Width} = \sqrt[4]{\frac{E_1}{E_2}}$ . A plate with such a ratio is called a 'Poisson test plate'

[4]. This name has been chosen based on the observation that the frequencies of the anticlastic and synclastic modes (figure 1) are particularly sensitive for changes of the Poisson's ratio of the plate material. A (hypothetical) material with a zero value for Poisson's ratio would make the frequencies of the anticlastic and synclastic mode coincide.

Figure 1 gives an overview of the five first mode shapes of a Poisson test plate. The mode shapes of the first three resonances (the torsional, anticlastic and synclastic modes) will always appear in this fixed sequence. The 4th and 5th frequencies coincide for a perfect Poisson plate. Due to inevitable small imperfections the order of the fourth and fifth mode, an orthogonal couple of torsion-bending mode shapes, can not be predicted a priori, and will have to be determined during the experiment.

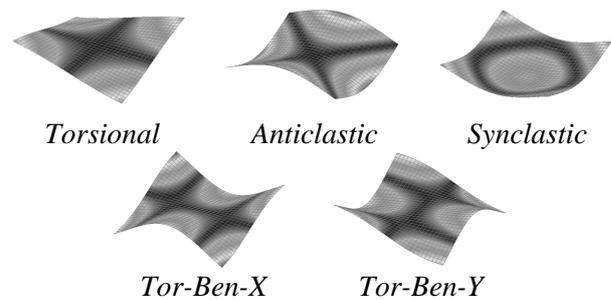


Figure 1: The first 5 mode shapes of a Poisson test plate.

### 2.2 Identification of the engineering constants

A detailed scheme of the Resonalyser procedure is given in figure 2. Starting with estimated initial values, the engineering constants in a finite element model of the test plate are iteratively updated until the first five computed resonance frequencies match the measured frequencies. In the finite element model, the plate dimensions and mass are considered as known quantities and thus fixed values. The four engineering constants are stored in the parameter column  $\{p\}$ . The updating of  $\{p\}$  is realized by minimizing a cost function  $C(p)$ :

$$C(p) = \left\{ f_{exp} - f_{FEM}(p) \right\}^T \left[ W^{(f)} \right] \left\{ f_{exp} - f_{FEM}(p) \right\} + \left\{ p^{(0)} - p \right\}^T \left[ W^{(p)} \right] \left\{ p^{(0)} - p \right\} \quad (2)$$

in which  $C(p)$  is a  $\mathbb{R}^{NP} \rightarrow \mathbb{R}$  cost function yielding a scalar value,  $NP=4$  is the number of

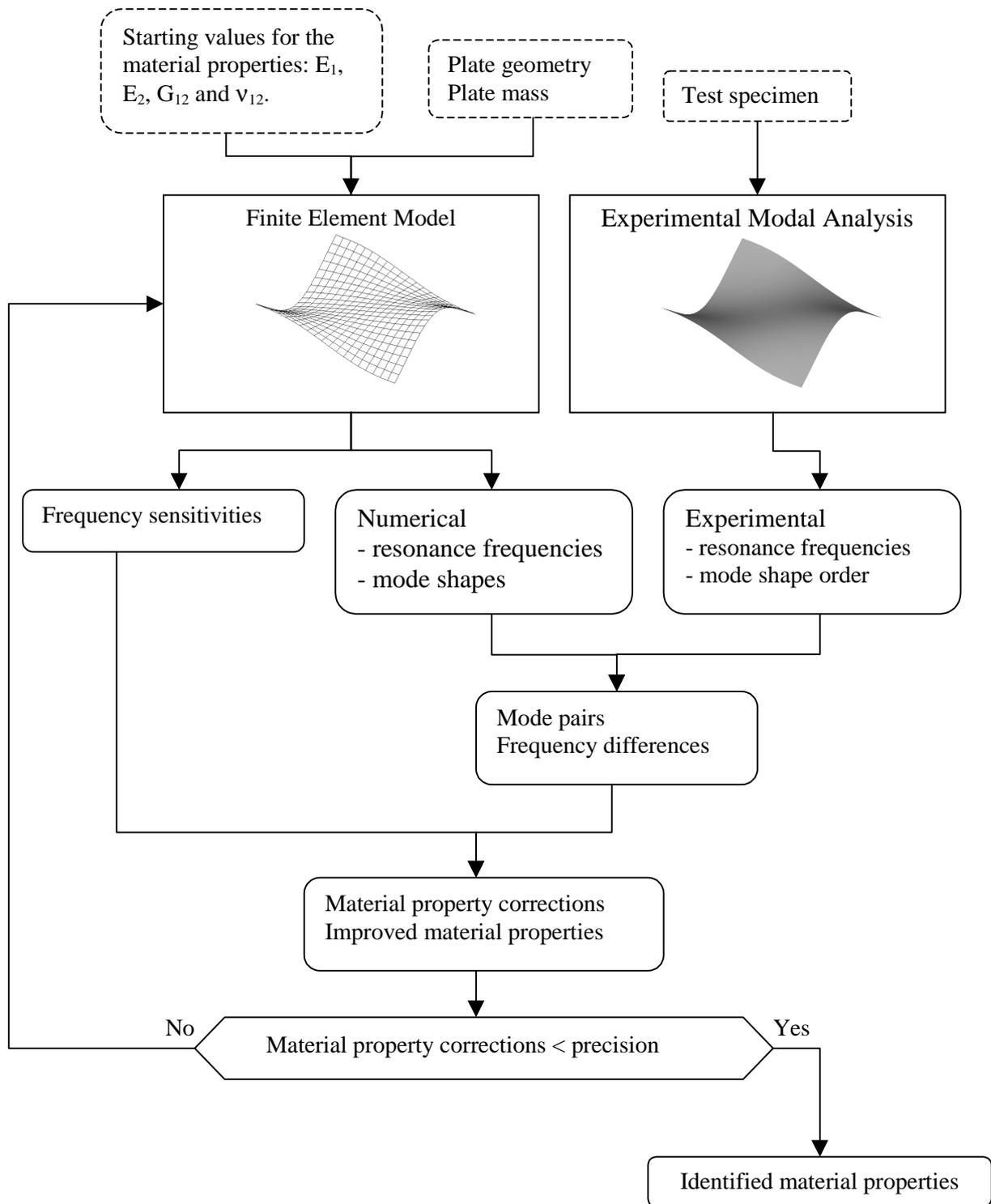


Figure 2: Detailed flowchart of the Resonalyser procedure: material identification by comparing the experimentally measured and computed resonance frequencies of a test plate.

material parameters:  $E_1$ ,  $E_2$ ,  $\nu_{12}$  and  $G_{12}$ ,  $\{p^{(0)}\}$  is a  $(NP \times 1)$  vector and contains the initial estimates for the material parameters,  $\{f_{FEM}(p)\}$  is a  $(NF \times 1)$  output column containing the  $NF=5$  computed frequencies using parameter values  $\{p\}$ ,  $\{f_{exp}\}$  contains the  $(NF \times 1)$  measured frequencies,  $[W^{(f)}]$  is a  $(NF \times NF)$  weighting matrix applied on the difference between the measured and the calculated frequency column,  $[W^{(p)}]$  is a  $(NP \times NP)$  weighting matrix for the difference between the initial parameter column  $\{p^{(0)}\}$  and the parameter column  $\{p\}$ .

The cost function  $C(p)$  has a minimal value for the optimal parameter values column  $\{p^{(opt)}\}$ . The value of this  $\{p^{(opt)}\}$  can be made independent on the choice of the weighting matrices  $[W^{(f)}]$  and  $[W^{(p)}]$  in the cost function. The choice and role of  $[W^{(f)}]$  and  $[W^{(p)}]$  is discussed, among others, in [2], [5] and [6]. The updating of the initial parameter column toward  $\{p^{(opt)}\}$  by minimization of the cost function is given by the following recurrence formula in iteration step  $(j+1)$ :

$$\{p^{(j+1)}\} = \{p^{(j)}\} + \left[ W^{(p)} + S^{(j)T} W^{(f)} S^{(j)} \right]^{-1} \times S^{(j)} W^{(f)} \{f_{exp} - f_{fem}(p^{(j)})\} \quad (3)$$

In (3)  $S$  is the sensitivity matrix containing the partial derivatives of the numerical frequencies with respect to the elements of the parameter column.

The numerical model of the test plate is based on the Love-Kirchhoff theory [7]. The applicability of this theory is mainly limited by the thickness of the plate. Traditionally, plates with a length/thickness ratio that exceeds a factor of 50 are considered as sufficiently thin. The tested materials and applied vibration amplitudes, do not violate additional assumptions made by the Love-Kirchhoff theory. Very accurate eight order polynomial Lagrange functions are taken as shape functions in the used numerical finite element model of the test plate [4]. The stiffness matrix of the test plate is evaluated in each iteration cycle using standard finite element procedures with the values of the parameter column  $\{p\}$  at that moment [8]. The computed resonance frequencies are obtained by solving a generalized eigenvalue problem composed with the constant mass matrix and the evaluated stiffness matrix [8]. The iteration procedure ends if convergence of  $\{p\}$  is reached. The values of the engineering constants in  $\{p\}$  after the last iteration cycle are considered as the result of the Resonalyser procedure. The whole identification procedure takes typically less than 5 seconds on a standard pentium-III PC.

### 3 Material Identification

#### 3.1 Test specimen selection and preparation

Commercially available 6082 Al-alloy, 304 stainless steel, and CuZn37 brassplates were obtained in the hot-rolled state. The 304 stainless steel is an austenitic steel with as main alloying elements chromium (18.5 %) and Ni (8.5 %). The plate used in this investigation has a thickness of 6 mm. The 6082 aluminium alloy contains as main alloying elements silicon (0.7 – 1.3 %), iron (0.5 %), Mn (0.4 – 1.0 %) and Mg (0.6 – 1.2 %). The plate used in this investigation has a thickness of 5 mm. The tested brass is a Cu (62 – 64 %), Zn (35 – 37 %) alloy and contains Ni (0.3 %), Fe (0.1 %), and Sn (0.1 %) as main alloying elements. The used plate had a nominal thickness of 5 mm.

Tensile, Resonalyser and/or IET test samples were produced from single plates of each material. Relatively large samples were cut and carefully machined (to within +/- 0.05 mm) to eliminate or at least reduce the effect of inaccurate sample dimensions on the calculated material properties. The thickness of the samples was not changed: the rolling surfaces were left untouched. The standard deviation of the thickness was 0.013 mm. IET and tensile tests were performed on samples cut along the rolling direction and at the following angles: -90°, -60°, -45°, -30°, 30°, 45°, 60° and 90°. Plates for the Resonalyser tests had sides parallel to and at 90° with the rolling direction.

#### 3.2 Standard mechanical tests

The elastic moduli were determined following standard IET and tensile test procedures. IET is based on the measurement of the fundamental flexure and torsion resonance frequencies of slender beam samples. Analytical equations, based on elastic beam theory, relate these frequencies to the Young's and shear modulus while assuming isotropic material behaviour. IET-tests were performed on rectangular beam-like samples of nominal size  $300 \times 24 \times 5$  mm for the aluminium,  $300 \times 30 \times 5$  mm for the brass, and  $300 \times 24 \times 6$  mm for the steel, using the RFDA apparatus (IMCE nv, Diepenbeek, Belgium) described in [9], following the guidelines provided by ASTM E 1876-99 and ENV-843-2.

Tensile tests were performed on flat dog-bone shaped samples of length 200 mm, 65 mm gauge length and gauge section of 63 mm<sup>2</sup> for the aluminium and 73 mm<sup>2</sup> for the steel samples. The strain was determined in a first instance using a clip-on extensometer (gauge length 50 mm). Afterwards, tensile tests were repeated on the same samples, now using bi-axial strain gauges, in order to identify Poisson’s ratio. For each test, the load was applied and removed periodically, with an increasing amplitude (3, 6 and 9 kN for the aluminum samples, and 2, 4 and 6 kN for the steel samples).

## 4 Experimental Results

### 4.1 Impulse Excitation Test

The Young’s modulus was calculated from the fundamental in plane bending frequency (IP-Bending). The shear modulus was calculated from the fundamental torsion frequency of the beam. Figure 3 and 4 show the obtained material properties for different orientations of the test beams.

### 4.2 Uniaxial tensile tests

The Poisson’s ratios were identified by means of bi-axial strain gauges, while the measurement of the Young’s moduli was performed with a clip-on extensometer. The values of the elastic moduli were derived from the stress-strain curves obtained during the loading phase of the test cycle. The measured material properties are plotted in figure 3 and 4.

### 4.3 Resonalyser tests

The material properties were obtained with the procedure described in paragraph 2. Table 1 and table 2 compare the measured resonance frequencies with the analytical frequencies of the Resonalyser’s FE-model.

	Freq. Exp.	Freq. Num.	Difference
Mode 1	177.00	176.76	0.13 %
Mode 2	267.50	267.36	0.05 %
Mode 3	339.94	339.79	0.04 %
Mode 4	465.35	465.88	-0.11 %
Mode 5	466.09	466.62	-0.11 %

Table 1: The obtained frequency match and material properties for the aluminum plate.

	Freq. Exp.	Freq. Num.	Difference
Mode 1	220.53	220.09	0.20 %
Mode 2	322.73	322.51	0.07 %
Mode 3	400.92	400.67	0.06 %
Mode 4	562.86	563.82	-0.17 %
Mode 5	576.96	577.86	-0.16 %

Table 2: The obtained frequency match and material properties for the steel plate.

The Resonalyser method only identifies directly material properties in the direction of the main axes. The off-axis elastic properties of an orthotropic material can be calculated with the following equations [10]:

$$\begin{aligned}
 \frac{1}{E_x} &= \frac{1}{E_1} \cos^4 \theta + \left( \frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) \sin^2 \theta \cos^2 \theta + \frac{1}{E_2} \sin^4 \theta \\
 \nu_{xy} &= E_x \left[ \frac{\nu_{12}}{E_1} (\sin^4 \theta + \cos^4 \theta) - \left( \frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}} \right) \sin^2 \theta \cos^2 \theta \right] \\
 \frac{1}{E_y} &= \frac{1}{E_1} \sin^4 \theta + \left( \frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) \sin^2 \theta \cos^2 \theta + \frac{1}{E_2} \cos^4 \theta \\
 \frac{1}{G_{xy}} &= 2 \left( \frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) \sin^2 \theta \cos^2 \theta \\
 &\quad + \frac{1}{G_{12}} (\sin^4 \theta + \cos^4 \theta)
 \end{aligned} \tag{4}$$

### 4.4. Comparison of the results

As opposed to the Resonalyser results, the off-axis properties identified with IET and tensile tests are directly measured. Figures 3 and 4 compare these results with the curves obtained from formula (4) and the Resonalyser procedure.

## 5 Discussion

The anisotropy of the aluminum material is clearly more pronounced (about 4% difference between minimum and maximum Young’s modulus) than that of the investigated stainless steel (less than 2%).

### 5.1 Correlation between the results of the different tests

For both the aluminum and steel material a good correlation between the results of the Young’s moduli in the two main directions obtained with the three different methods is found. However, in the case of the aluminum plate, the Resonalyser procedure results in lower off-axis elastic moduli

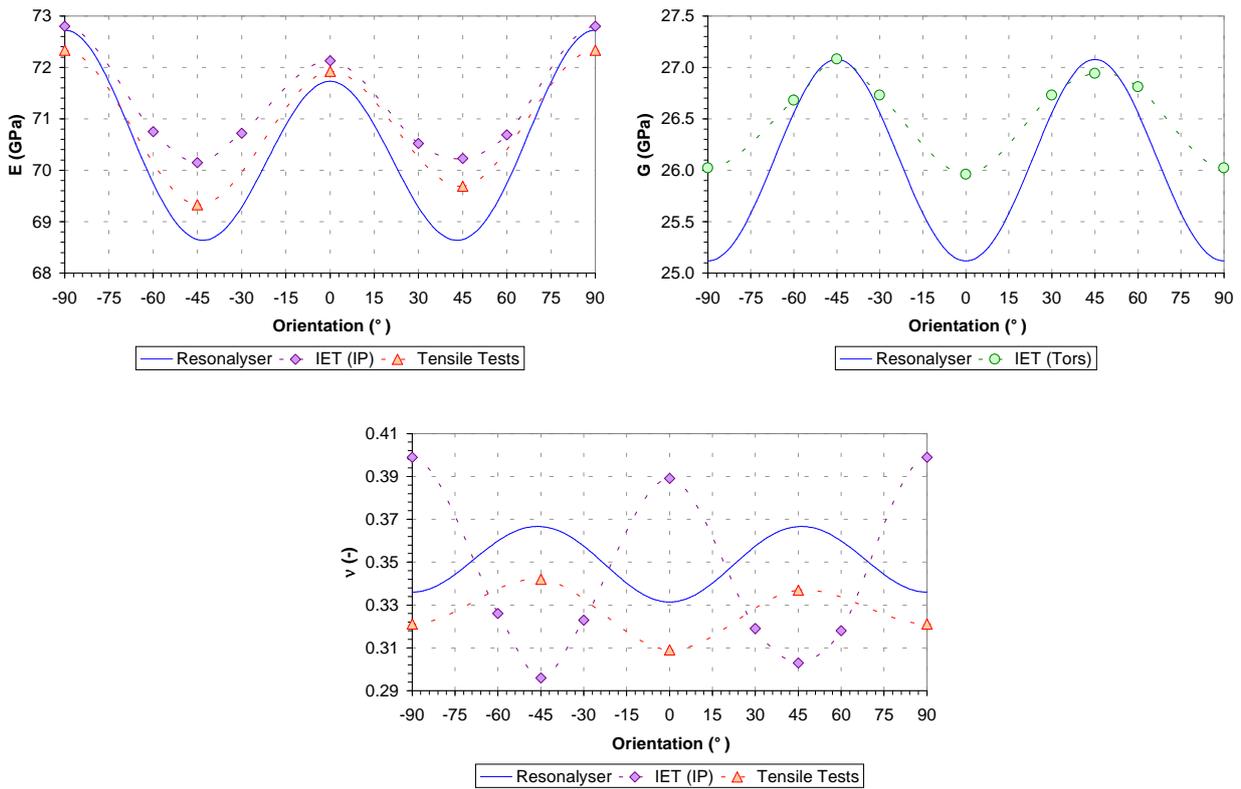


Figure 3: Comparison of the results of the three different methods for the E-modulus, G-modulus and Poisson's ratio  $\nu$  for aluminum.

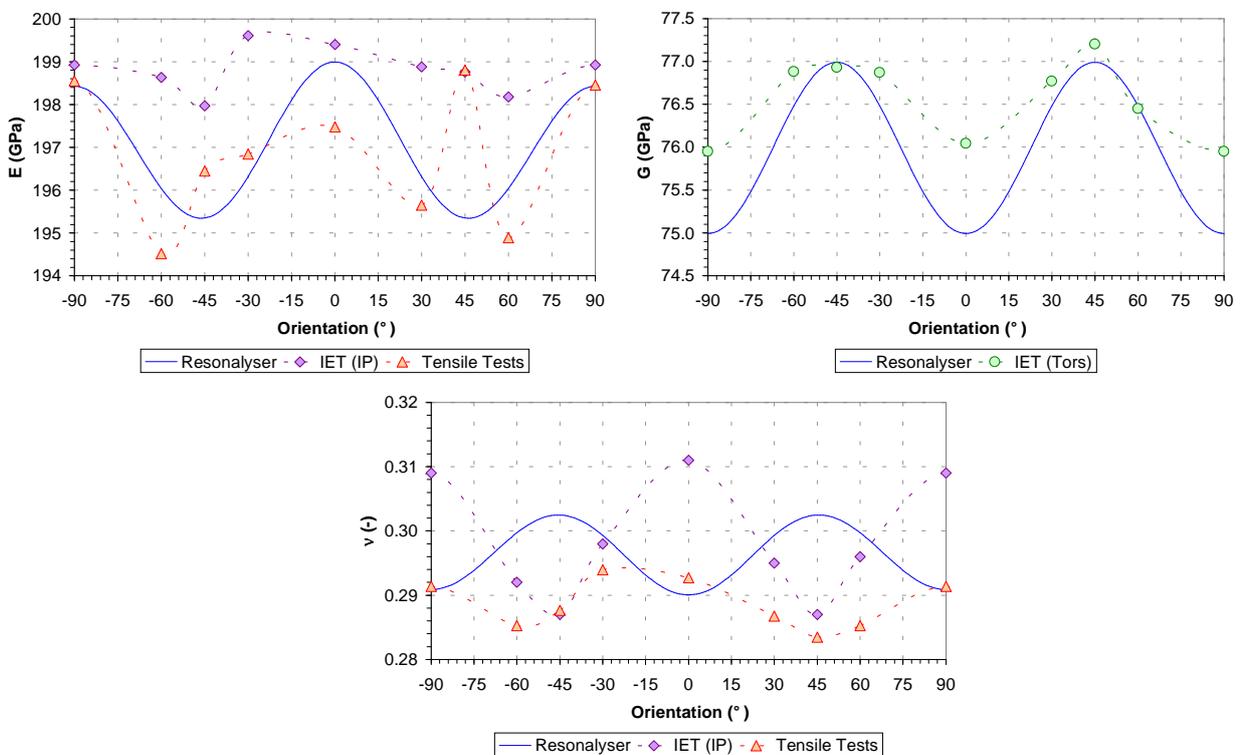


Figure 4: Comparison of the results of the three different methods for the E-modulus, G-modulus and Poisson's ratio  $\nu$  for steel.

than IET. The tensile tests performed on the specimen in the  $\pm 45^\circ$  directions yielded values that were situated between the results of the two dynamic test methods. The spreading on the results of the tensile tests was too high to indicate if IET overestimated or if the Resonalyser method underestimated the off-axis elastic moduli. The steel material was too isotropic to compare the difference between the correlation of the on-axis and off-axis Young's moduli.

For both the aluminum and steel material the values of shear modulus  $G_{12}$  obtained with IET are higher than the values obtained with the Resonalyser method, but there is a good correlation between the values of the shear moduli in the  $\pm 45^\circ$  directions.

The comparison between the Poisson's ratios obtained with IET and Resonalyser reveals a complete lack of agreement between the results of these two methods. However, the tensile tests and the Resonalyser procedure reveal the same directional dependency of Poisson's ratio, for the aluminum. IET fails to identify Poisson's ratio for orthotropic materials, since the IET identification-procedure is based on a relation between Poisson's ratio and the elastic and shear modulus, which only holds for fully isotropic materials.

### 5.2 Verification

To verify whether the observed differences of the Young's moduli in the off-axis directions and shear moduli in the on-axis directions are specifically related to the used aluminum material, rather than a more general problem, the Resonalyser method and IET were used to identify the properties of the brass material.

Three IET beams in the  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  directions and one Resonalyser plate ( $300 \times 300 \times 5$  mm) were machined. The brass samples were tested in the same way as the aluminum and steel samples.

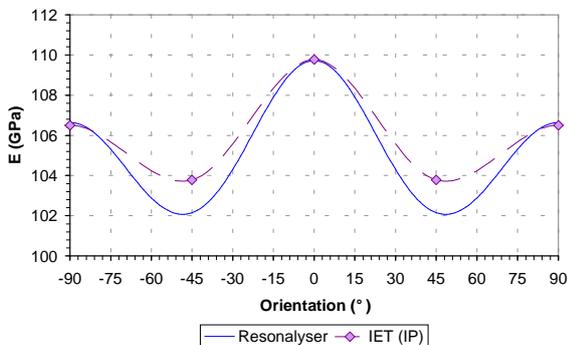


Figure 5: Comparison of the Young's moduli obtained with the Resonalyser and IET for the brass material.

The obtained Young's moduli are presented in figure 5. Once again, a good correlation between the on-axis properties is found. As for the aluminum, the Young's moduli found by IET are higher than those found by the Resonalyser procedure. Like in the previous tests, the shear modulus in the zero direction obtained with IET exceeds the shear modulus obtained with the Resonalyser method, figure 6.

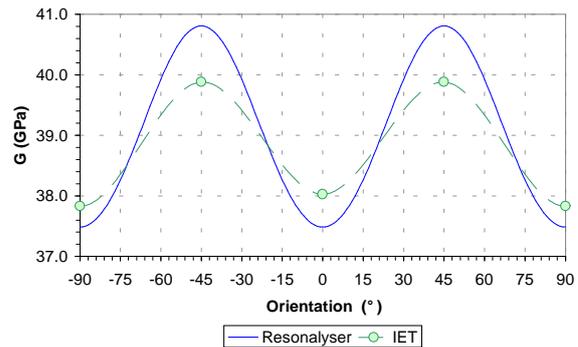


Figure 6: Comparison of the shear moduli obtained with the Resonalyser and IET for the brass material.

### 5.3 The influence of warping

The Kirchhoff plate theory, used in the Resonalyser's FE-model, assumes that a plane perpendicular to the central plane of the plate before deformation, remains flat and perpendicular to the central plane after deformation. The Kirchhoff thin plate theory does not account for warping deformations caused by shear stresses induced by torsion. For a given set of material properties, this could lead to an overestimation of the resonance frequency of all the torsion modes. The Resonalyser procedure will compensate this effect by artificially reducing the shear modulus. This is exactly what is observed when comparing the IET and Resonalyser results. This could explain why the shear modulus found with the Resonalyser is lower than the shear modulus found with IET. An increase of the shear modulus will also lead to an increase of the off-axis elastic properties calculated with (4).

Further research will be needed to confirm whether warping deformation is really causing the observed differences between the results of the two dynamic measurement methods.

## 6 Conclusion

In this paper three methods for measuring the elastic properties of plate materials are shown to quantify the degree of elastic anisotropy of aluminum and stainless steel sheets. The results obtained with different techniques (uniaxial tension, impulse excitation technique and a mixed-numerical-experimental technique) confirm each other.

Small differences between the off-axis elastic moduli obtained with the Resonalyser and IET technique were observed. The influence of warping is proposed as an explanation for these differences. Further research is needed to prove the validity of this theory.

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