Validation of a vibration based identification procedure for layered materials.

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Abstract

Layered materials are becoming increasingly important for the production of high performance components and constructions. Their stiffness properties are fundamental to assess stress fields during design calculations. A testing method to assess the in-plane elastic properties of each individual layer of a laminate has been developed [1]. The proposed procedure is based on a multi-model updating routine, in which the elastic properties of the finite element models of the test samples are simultaneously updated. Once the finite element models reproduce the measured resonance frequencies, the updating procedure is halted, and the elastic properties of the different layers can be retrieved from the finite element model's database. This paper focuses on the experimental validation of this new measurement procedure. The validation is carried out by measuring the elastic properties of a reference material, i.e. a layered material of which the elastic properties of the layers are known. The used reference material was obtained by gluing a stainless steel sheet to a brass sheet.

1 Introduction

Elastic material properties play a major role in the vibratory behavior of structures. Vibration based material identification methods are founded on this fundamental relation. These methods derive the elastic material properties from the vibratory behavior of the test sample. One of the first applications of this approach was reported in 'Zeitung Für Metallkunde' by Förster in 1937 [2]. Förster used the Euler beam theory to link the elastic modulus to the eigenfrequency of the specimen's fundamental flexural mode. His method has been refined by Pickett [3], Spinner and Teft [4]. The work of Spinner and Teft formed the base of the current ASTM resonant beam test procedure [5], which standardized vibration testing based on analytical relations.

The use of analytical formulas to describe the vibratory behavior of test specimens is however a major obstacle for extending the vibration based methods to more complex materials. In 1986, Sol [6] replaced the analytical formulas by special purpose finite element models. The derived identification method, which is called the 'Resonalyser' method [7], can simultaneously identify the four engineering constants of an orthotropic material – i.e. E_1, E_2, G_{12} and ν_{12} – from the experimental resonance frequencies of one single test plate. In 2003 this method was extend to layered materials [1], in such a way that it became possible to identify the orthotropic elastic properties of each layer of a laminate.

2 Vibration based material identification

2.1 Mixed numerical-experimental approach

As shown in [6], mixed numerical-experimental techniques (MNETs) can be used to identify elastic material properties. Figure 1 displays the general flowchart of the MNET procedure. The experimental part consists of a modal analysis test performed on freely suspended test samples. This test configuration is used because it can be approximated by free-free boundary conditions in the finite element model. The measured resonance frequencies are used as input data for the identification routine. The numerical part of the method consists of a highly accurate finite element model. The numerical frequencies are calculated using a set a trial values for the unknown material parameters. The numerical frequencies are compared with the measured frequencies, and corrected material properties are found by minimizing the output risiduals. The output risiduals are frequency differences between the experimental and numerical frequencies. The improved material properties are inserted in the FE-model and a new iteration cycle is started. Once the numerical and experimental frequencies match, the procedure is aborted, and the desired material properties can be found in the database of the finite element model. Numerous test cases like [8], [9], [10] and [11] proof that MNETs are a reliable tool to identify elastic material properties.

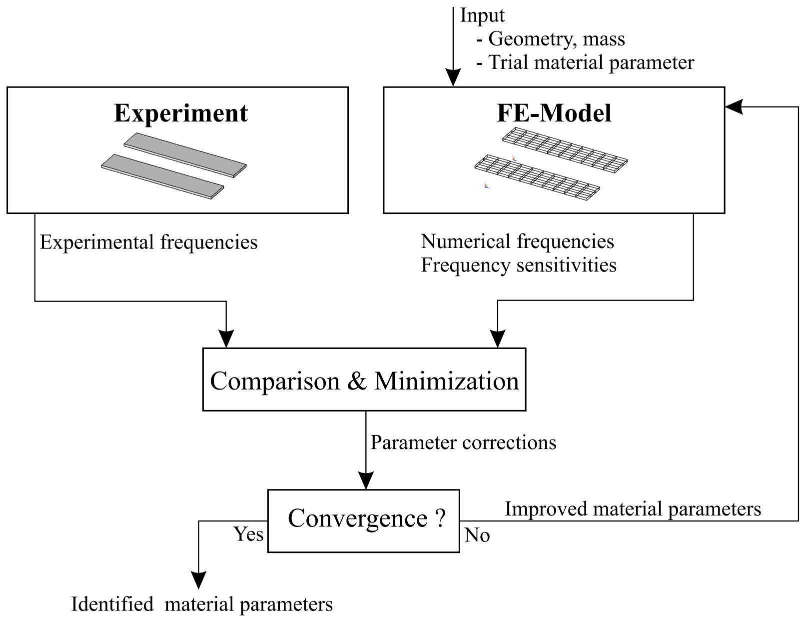


Figure 1: General flowchart for MNET based elastic material identification procedure.

The MNET procedure contains an optimization problem of which the objective function can be obtained as follows. A sensitivity analysis performed on the finite element model provides a linearized relation between an imposed parameter change and the resulting frequency shift in a particular working point of the parameter space.

$$[S]\{\Delta p\} = \{\Delta f\} \tag{1}$$

The vectors $\{\Delta p\}$ and $\{\Delta f\}$ contain the parameter and frequency changes, respectively. The matrix $[S]$ is the sensitivity matrix and contains the frequency sensitivities with respect to the material parameters. The elastic properties are found by minimizing the frequency differences between the experimental and numerical frequencies $\{\Delta r\}$. The optimal frequency shift is thus defined by

$$\underset{\Delta f}{minimize} \left\| \{\Delta f\} - \{\Delta r\} \right\|_2^2 \tag{2}$$

where $\| \diamond \|_2$ denotes the Euclidean norm. Inserting the sensitivity relation (1) into (2) provides the mathematical expression of the MNET's optimization problem.

$$\underset{\Delta p}{minimize} \left\| [S]\{\Delta p\} - \{\Delta r\} \right\|_2^2 \tag{3}$$

To ensure a stable convergence of the iterative procedure, the optimization problem (3) is solved by considering a set of box constraints on the optimization parameters – the elements of the vector $\{\Delta p\}$ – in such a way that each material parameter cannot change more than 25% during one iteration step.

2.2 Single-beam versus multi-beam procedure

The sensitivity analysis, necessary to obtain the sensitivity matrix $[S]$, can be executed in the global or in a local axis system. Figure 2 introduces both the global or material axis system – 1-2 – and the local axis system – x-y.

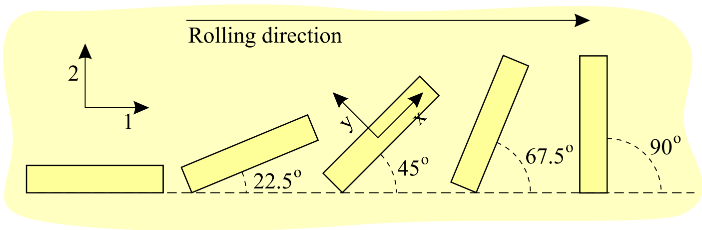


Figure 2: A typical sample set for the multi-beam identification routine.

The global axis system is defined by the principal material directions. In the case of sheet metal, the principal directions are the directions parallel and perpendicular to the rolling direction. In this axis system, the behavior of an orthotropic material can be described with four independent parameters: E_1, E_2, G_{12} and ν_{12}. The local axis system is aligned to the edges of the samples. In this axis system, the properties E_x, E_y, G_{xy} and ν_{xy} are no longer independent, and represent the apparent elastic properties in the x- and y-direction.

Figure 3 shows the results of a sensitivity analysis of the first five frequencies of a beam-shaped brass specimen. The sensitivity analysis was performed in the local coordinate system. The first, second and fourth vibration mode are bending modes, and their resonance frequencies appear to be solely determined by the apparent elastic modulus E_x. The torsion frequencies, modes three and five, are only sensitive to changes of the apparent shear modulus G_{xy}. These results lead to two major conclusions: a) all the bending/torsion frequencies contain the same information, it is thus not necessary to use more than one bending/torsion frequency, b) it is possible to identify the apparent elastic and shear modulus from the resonance frequencies of one single beam-shaped specimen. Because this approach uses the frequencies of just one beam, it will be referred to as the 'single-beam identification' procedure. An estimate of Poisson's ratio can be derived from

$$G = \frac{E}{2(1+\nu)} \tag{4}$$

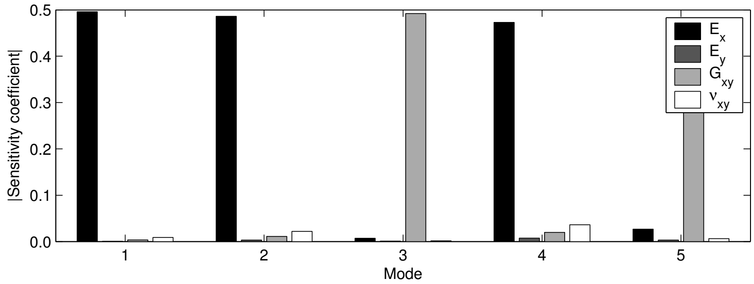


Figure 3: Sensitivity analysis in local material coordinates.

The obtained Poisson's ratio will only be correct if the material is elastically isotropic. The variation of the material properties in function of the material orientation can be obtained by measuring a set of beams that each represent a different material direction, e.g. the sample set of figure 2.

But the single beam identification procedure is not a very intelligent approach. It does not consider any relations between the material properties that are obtained in the various material directions. For an orthotropic material, all these properties are related to the principal material parameters as:

$$\frac{1}{E_x} = \frac{1}{E_1}\cos^4\theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\sin^2\theta\cos^2\theta + \frac{1}{E_2}\sin^4\theta \tag{5}$$

$$\frac{1}{E_y} = \frac{1}{E_1}\sin^4\theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)\sin^2\theta\cos^2\theta + \frac{1}{E_2}\cos^4\theta \tag{6}$$

$$\frac{1}{G_{xy}} = 2\left(\frac{2}{E_1} + \frac{2}{E_2} - \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta + \frac{1}{G_{12}}\left(\sin^4\theta + \cos^4\theta\right) \tag{7}$$

$$\frac{1}{\nu_{xy}} = E_x\left[\frac{\nu_{12}}{E_1}\left(\sin^4\theta + \cos^4\theta\right) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)\sin^2\theta\cos^2\theta\right] \tag{8}$$

It is possible to incorporate these relations into the identification procedure by performing the sensitivity analysis in the global coordinate system. Figure 4 gives the results of such a sensitivity analysis for the five brass beams of figure 2. For each beam, only the fundamental bending and torsion frequency were taken into consideration. The results show a good sensitivity for every global material parameter. So, by considering the resonance frequencies of a whole set of beams, it is possible to directly identify the pricipal orthotropic material parameters E_1, E_2, G_{12} and ν_{12}. Because the frequencies of a set of beams are being used, the MNET will have to simultaneously update a number of finite element models. In the rest of the text this approach will be referred to as the 'multi-beam identification' procedure. Note that the mulit-beam approach requires a minimum of two beams, of which at least one is not oriented in a direction parallel to the principal material directions.

Since all the bending and torsion frequencies contain the same information, and since finite element models converge faster for lower order modes, it is preferred to use only the fundamental bending and torsion frequencies as experimental input data for the MNET identification routines.

2.3 Layered materials

Due to the flexibly of finite element models, MNETs are an obvious choice to develop test procedures to measure the elastic properties of layered materials. The extension of non-layered to layered identification routines is discussed in detail in [1]. This article explaines why it is impossible to identify the elastic properties of the layers of a laminate from the frequencies of one single specimen. Resonance frequencies of

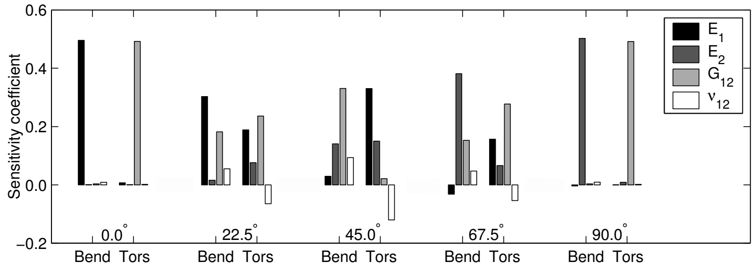


Figure 4: Sensitivity analysis in global material coordinates.

laminates appear to be controlled by the overall stiffness of the material. The same overall stiffness can be obtained with a infinite number of different layer stiffness combinations. Therefore, it is impossible to decompose the overall stiffness into the correct layer stiffnesses, since there will be an infinite number of solutions. This uniqueness problem can be overcome by using the resonance frequencies of a set of test specimen. Each specimen of this set must be made with the same material layers, but must have a different overall stiffness. [1] shows that there will only be one set of layers stiffnesses that will result in the correct overall stiffness for all the test specimen, if the number of test specimen is larger or equal to the number of unknown material layers. In this way, the unicity of the solution can be assured.

The overall stiffness of the laminate can only be altered by changing the stacking sequence of the layers or by changing the thickness of – one of – the layers, which means that layered material identification requires a number of purpose built samples. When these purpose built samples are not available, they can be obtained by – partially – removing a layer. But removing a part of the material is not an optimal approach, since this will most likely damage the sample, e.g. cracks, delamination. An alternative approach is to add a layer to the laminate. If the properties of this layer are measured before it is added to the sample, it is possible to produce an additional layer configuration without introducing any new unknown parameters. A candidate material for this additional layer is glass. Glass is preferred because is very homogeneous, it is cheap, it can be glued to almost any material and it is transparent. The transparency of glass has two major advantages, a) it allows to visually check the quality of the glue layer, e.g. the presence of air bubbles, b) it allows the use of UV-curing glue. UV-curing glue is very convenient for this purpose because it gives the user plenty of time to correctly position the additional layer on the laminate and to remove possible air bubbles, in a combination with the very short curing time.

Both the single-beam and multi-beam identification approaches can easily be extended to layered materials, by considering all the used samples configurations in the updating procedure of the MNET.

2.4 The sensitivity matrix

Sensitivity matrices group sensitivity coefficients. Traditionally, these sensitivity coefficients are defined as the partial derivatives of a model response with respect to a model parameter or in the case of material identification

$$s_{ij} = \frac{\partial f_i}{\partial p_j} \tag{9}$$

The use of these absolute sensitivities is not advisable to identify elastic material parameters. Absolute sensitivities can results in ill-conditioned sensitivity matrices because their value depends on the absolute value of the model parameters. When the model parameters differ several orders of magnitude, e.g. elastic moduli and Poisson's ratios, the sensitivity coefficients will too. Absolute sensitivities also result in a minimization the absolute frequency differences instead of the relative frequency differences, indirectly increasing the importance of the higher order modes. The two problems can be solved by using the relative normalized sensitivity coefficients defined by (10).

$$s_{ij} = \frac{\partial f_i}{\partial p_j}\frac{p_j}{f_i} \tag{10}$$

The methods discussed above require the simultaneous updating of a set of finite element models. For each model, a sensitivity analysis provides:

$$\{\Delta f\}_k = [S]_k\{\Delta p\} \tag{11}$$

in which $[S]_k$ is the sensitivity matrix of the k^{th} sample (configuration), and $\{\Delta f\}_k$ the frequency shift caused by the parameter change $\{\Delta p\}$. The global sensitivity matrix $[S]$ groups the sensitivity matrices of the individual models.

$$\underbrace{\begin{Bmatrix}\Delta f_1\\ \Delta f_2\\ \vdots\\ \Delta f_n\end{Bmatrix}}_{\{\Delta f\}} = \underbrace{\begin{bmatrix}S_1\\ S_2\\ \vdots\\ S_n\end{bmatrix}}_{[S]}\{\Delta p\} \tag{12}$$

in which $\{\Delta f\}$ is the global frequency shift vector. The optimization problem of a multi-model MNET is obtained by inserting expression (12) into (3).

The structure of the model sensitivity matrices $[S]_k$ depends on the identification routine. In the case of the single-beam identification routine, the sensitivity matrix contains the sensitivities of the bending and torsion frequency with respect to the apparent elastic and shear moduli of the different material layers or

$$[S]_k = \begin{bmatrix} \overbrace{\begin{matrix}\frac{\partial f_b}{\partial E_{x,1}}\frac{E_{x,1}}{f_b} & \frac{\partial f_b}{\partial G_{xy,1}}\frac{G_{xy,1}}{f_b}\\ \frac{\partial f_t}{\partial E_{x,1}}\frac{E_{x,1}}{f_t} & \frac{\partial f_t}{\partial G_{xy,1}}\frac{G_{xy,1}}{f_t}\end{matrix}}^{Material\,1} & \cdots & \overbrace{\begin{matrix}\frac{\partial f_b}{\partial E_{x,n_m}}\frac{E_{x,n_m}}{f_b} & \frac{\partial f_b}{\partial G_{xy,n_m}}\frac{G_{xy,n_m}}{f_b}\\ \frac{\partial f_t}{\partial E_{x,n_m}}\frac{E_{x,n_m}}{f_t} & \frac{\partial f_t}{\partial G_{xy,n_m}}\frac{G_{xy,n_m}}{f_t}\end{matrix}}^{Material\,n_m} \end{bmatrix}_k \tag{13}$$

in which f_b and f_t are the bending and torsion frequency, respectively. The modulus $E_{x,i}$ is the apparent elastic modulus in the x-direction of the i^{th} material. In the case of the multi-beam identification techniques the model sensitivity matrices contain the partial derivatives of the frequencies with respect to the four principal orthotropic elastic parameters.

$$[S]_k = \begin{bmatrix} \overbrace{\begin{matrix}\frac{\partial f_b}{\partial E_{1,1}}\frac{E_{1,1}}{f_b} & \cdots & \frac{\partial f_b}{\partial \nu_{12,1}}\frac{\nu_{12,1}}{f_b}\\ \frac{\partial f_t}{\partial E_{1,1}}\frac{E_{1,1}}{f_t} & \cdots & \frac{\partial f_t}{\partial \nu_{12,1}}\frac{\nu_{12,1}}{f_t}\end{matrix}}^{Material\,1} & \cdots & \overbrace{\begin{matrix}\frac{\partial f_b}{\partial E_{1,n_m}}\frac{E_{1,n_m}}{f_b} & \cdots & \frac{\partial f_b}{\partial \nu_{12,n_m}}\frac{\nu_{12,n_m}}{f_b}\\ \frac{\partial f_t}{\partial E_{1,n_m}}\frac{E_{1,n_m}}{f_t} & \cdots & \frac{\partial f_t}{\partial \nu_{12,n_m}}\frac{\nu_{12,n_m}}{f_t}\end{matrix}}^{Material\,n_m} \end{bmatrix}_k \tag{14}$$

3 The validation test

3.1 An experimental verification

The layered identification method discussed in the previous section has already been validated with a number of numerical experiments. Those numerical validations were carried out using a set of virtual test samples.

The experimental frequencies of these samples were obtained by calculating them with finite element models. The obtained resonance frequencies were inserted into the layered identification routine to extract to elastic layer properties. This approach does not only have the advantage that no actual test samples have to be machined, it is also very convenient for validation purposes since the 'unknown' material properties are in fact known, i.e. the elastic properties of the finite element models used to generate the experimental frequencies.

However, a full verification of the proposed identification methods should also include an experimental validation. The main problem with an experimental validation is absence of an absolute reference. The elastic properties of real test samples are not known, and they cannot be easily measured since there are no standardized testing procedures for layered materials. A usefully validation test thus requires a special layered material, i.e. a layered material of which the materials of the different layers also are available in homogeneous – non-layered – form. In this way the material properties of the layer materials can be measured on the homogeneous samples, using standardized test. These values can then be used as reference values to estimate the accuracy of the layered identification method, by comparing them with the properties obtained on the laminate. For this validation test a purpose build bi-metal was used. This bi-metal was made up with a steel layer and brass layer with a nominal thickness 0.6 mm and 0.8 mm,respectively.

3.2 Sample preparation

With a water jet cutting technique, a set of test samples was cut from two large steel and brass plates. For each metal, the set of test samples comprised a series of 140 beams – 100×20mm – that were cut out in the 0.0°, 22.5°, 45.0°, 67.5° and 90.0° directions. These directions indicate the angle between the samples long axis and the rolling direction of the metal sheets, figure 2. The two metal layers were connected by gluing them with a two component epoxy glue, Permabond Bondmaster E32 general purpose toughened structural epoxy [12]. To achieve a glue layer which is as thin as possible, the samples were subjected to a pressure of approximately 40 bar during the first phase of the curing cycle, which took about 8 hours. With this process, glue layers with a thickness ranging between 5 to 15 μm could be obtained. Once the glue was fully cured, the sample edges were finished with a sanding process. This was necessary because it is impossible to keep the two metal layers perfectly aligned during the gluing process. Figure 5 schematically represents the result of the sanding process.

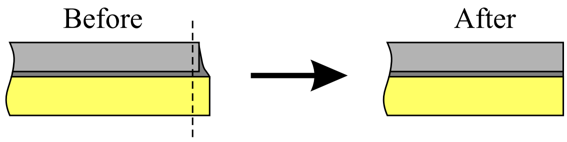


Figure 5: The effect of the sanding process.

This reference material is made up with two material layers, which means that the application of the proposed identification technique requires at least two samples configurations. The first configuration was the initial layered material – the steel-brass configuration. The second configuration was obtained by adding a glass layer to the bi-metal. The additional glass layers were glued to metal samples with the Bohle B-682-0 UV-curing glue [13]. To improve the quality of the edges the samples were once again sanded after adding the glass layer. To examine the importance of the thickness of the glass layer on the obtained results, glass sheets with a thicknesses of 0.5 mm and 0.7 mm were used.

The pure steel – brass – samples were obtained by gluing two steel – brass – layers. The gluing process was necessary to obtain flat samples since the initial metal sheets were lightly curved. Curved samples have to be avoid because the curvature will increase the stiffness of the samples. If this curvature is not modeled in the finite element models of the identification routine, the induced stiffness increase will be attributed to the elastic material parameters, and will thus result an overestimation of the material properties. Flat samples were preferred over the introduction of the curvature into the identification models, because the latter would increase the complexity and uncertainty of the validation test, and subsequently complicate the interpretation of the obtained results.

One hundred and forty beams, equally divided over steel and brass, were cut out of the metal sheets. For each metal, the set of seventy beams contained fourteen beams in each of the five considered material orientations. These hundred and forty beams were used to procedure thirty layered – brass-steel –, twenty homogeneous steel – steel-steel – and twenty homogeneous brass – brass-brass – beams, equally spread over the five material orientations. The thirty layered beams were modified with two different types of glass. Twenty of the layered beams, four samples for each material orientation, were modified by adding a 0.7 mm glass layer. Ten layered beams were altered by adding the glass the steel side, the other ten were modified by gluing the glass to the brass side. Ten layered beams, two samples in each material direction, were altered by adding a 0.5 mm glass layer. One half of these samples was made by adding the glass to steel side, for the other half of the samples the glass was glued to the brass side. Figure 6 gives on overview of the layer sequences of all these sample configurations.

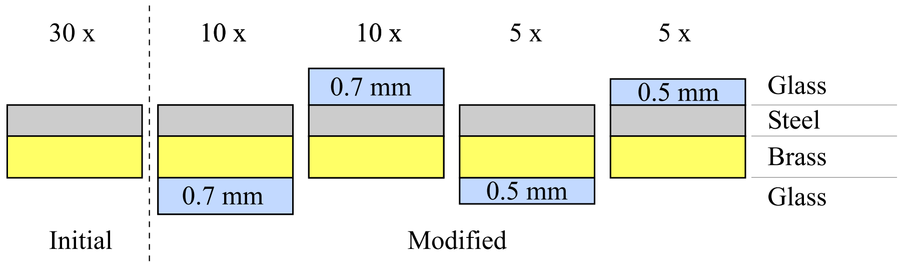


Figure 6: The different configurations of the test samples.

4 The obtained results

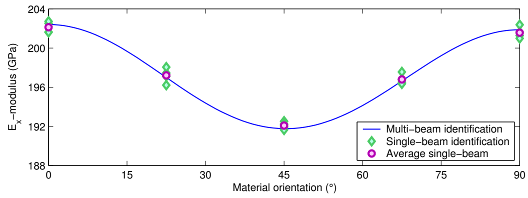
4.1 Non-layered samples

The fundamental bending and torsion frequencies of the homogeneous steel and brass samples were measured and their elastic properties were estimated with both the single- and multi-beam identification procedures. Figure 7 presents the results obtained on the steel beams, figure 8 presents the properties of the brass beams. For the two materials, the correlation between the properties found with the single- and multi-beam method are excellent for the elastic and shear modulus. However, for Poisson's ratio there is absolutely no agreement between the results obtained with the two methods. The graphs of the elastic and shear modulus clearly indicate that these two properties vary in function of the orientation. This means that the both the steel and brass material have orthotropic elastic properties. In the case of the single-beam identification routine, the value of Poisson's ratio is derived from relation (4) which is only valid for isotropic materials. The Poisson's ratios obtained with the single-beam identification routine are thus incorrect.

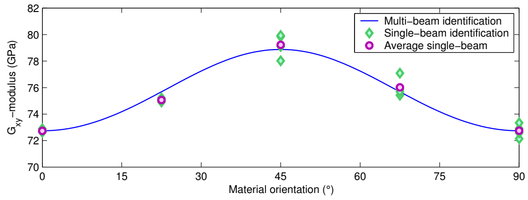
4.2 Layered samples

The fundamental bending and torsion frequencies of the layered samples were measured two times, once for the initial steel-brass configuration, and once for the steel-brass-glass configuration. Before the glass samples were added to the layered test specimen, their properties were tested with the single-beam identification technique. During the layered identification routines, the properties of the glass layers will be kept fixed.

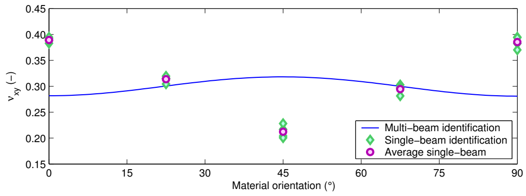
In a first phase the properties of the layered samples were identified with the single-beam approach. Note that in the case of the considered layered material, the single-beam technique uses the frequencies of two sample



(a) The elastic modulus

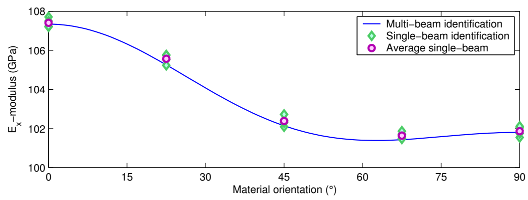


(b) The shear modulus

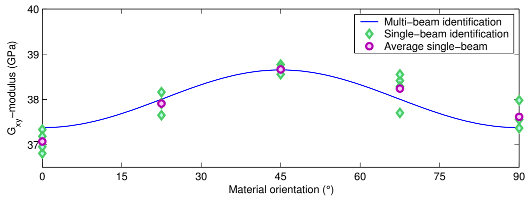


(c) Poisson's ratio

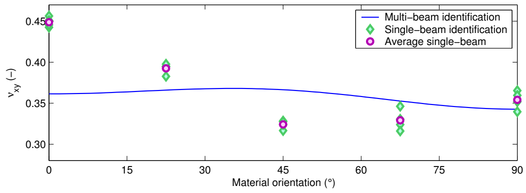
Figure 7: The properties of the homogeneous steel beams.



(a) The elastic modulus



(b) The shear modulus

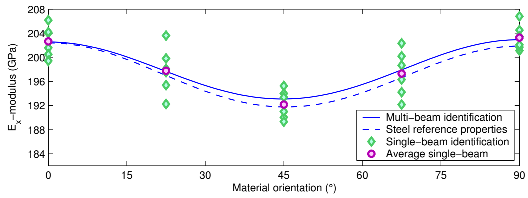


(c) Poisson's ratio

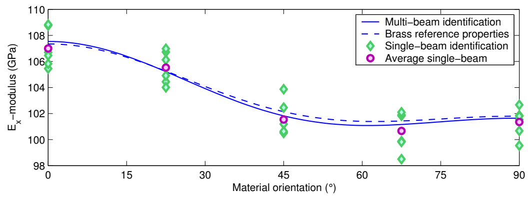
Figure 8: The properties of the homogeneous brass beams.

configurations of the same beam, and thus requires two finite element models. The diamonds in the plots of figure 9 represent the results of the single-beam identification routine. The purple dots indicate the average values of the obtained layer properties. The data points that are marked with a cross in figure 9 were not used in the calculation of the average properties. These six shear modulus values were rejected because they lay to far away from the other data points. The correct properties of the layer materials, i.e. the properties obtained on the homogeneous metal specimen, are represented by the dashed lines. The spreading of the results obtained on the layered samples in higher that the spreading obtained on the homogeneous samples. But the average values of the elastic and shear modulus are good, both for the steel and brass layers. The average difference between the properties obtained on the layered samples and the reference values is 0.4$\%$ for $E_{x,steel}$, 0.9$\%$ for $G_{xy,steel}$, 0.5$\%$ for $E_{x,Brass}$ and 0.6$\%$ for $G_{xy,brass}$. The results for Poisson's ratio are incorrect, but this is related to a problem with the single-beam identification routine, and has nothing to do with the proposed identification approach for layered materials.

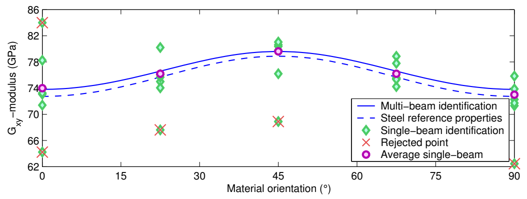
In a second phase the properties of the layered samples were identified with the multi-beam approach. Since there are thirty test samples, and two samples configurations for each samples, the multi-beam approach required the simultaneous updating of sixty finite element models. The torsion frequencies associated with the rejected shear modulus values were not used in the multi-beam identification. The solid lines of figure 9 represent the multi-beam identification results . The results obtained on the layered samples, are similar to those obtained on the homogeneous samples, even for Poisson's ratio. The average difference between the reference values and the properties identified on the layered samples are 0.5% for $E_{x,steel}$, 1.2% for $G_{xy,steel}$, 3.1% for $\nu_{xy,steel}$, 0.2% for $E_{x,Brass}$, 0.2% for $G_{xy,brass}$ and 0.3% for $\nu_{xy,brass}$.



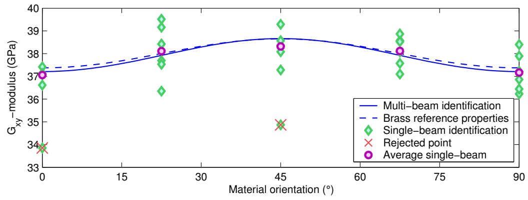
(a) The elastic modulus - steel layer



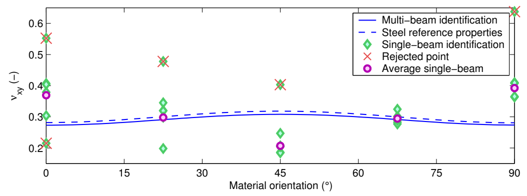
(b) The elastic modulus - brass layer



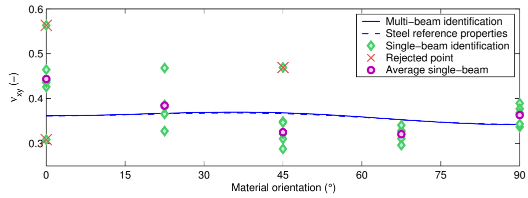
(c) The shear modulus - steel layer



(d) The shear modulus - brass layer



(e) Poisson's ratio - steel layer



(f) Poisson's ratio - brass layer

Figure 9: The material properties obtained on the layered samples.

5 Conclusions

A vibration based mixed numerical-experimental technique to identify the elastic material properties of the individual layers of layered material was experimentally validated. The validation was performed by identifying the properties of a bi-metal of which the layer properties were known. In this way the results obtained on the layered material could be compared with the correct material properties. The validation test has shown that the proposed approach is capable of correctly identifying the layer properties from the resonance frequencies of a set of beam-shaped samples. The single-beam routine can identify the elastic and shear modulus of the constitutive layers. The multi-beam routine can identify the full orhtoropic material parameter set of each layer.

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